

# Beyond Cash: Venture Capital, Firm Dynamics, and Economic Growth\*

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**\*\*\* PRELIMINARY – COMMENTS WELCOME \*\*\***

## Abstract

This paper presents a new dynamic general equilibrium model of innovation with heterogeneous firms that incorporates an explicit venture capital (VC) market. The data show that VC financing accounts for a disproportionate share of sales and employment in the US compared with its limited share of total investment. VC firms invest heavily in young and innovative firms, bringing *operational knowledge*, together with financing, to their portfolio companies. The goal of this paper is twofold. First, I measure the particular channels through which VC firms influence their undertakings, using a structural model. Second, I explore the implications of VC investments for aggregate productivity and innovation policy. To address these goals, I combine and structurally estimate an endogenous technical change model with a VC setting that includes (i) the new feature of expertise, and (ii) the endogenous matching market where firms and VCs meet. In this model, firms improve the quality of their innovative product through risky R&D. VC expertise raises the efficiency of product development, and firms obtain VC financing at the cost of selling an endogenously determined share of the company. The entry cost that VC companies face also introduces a selection margin: VCs invest in firms that present a high potential for growth. The estimated model captures certain features of the VC matches and innovation observed in the US data. Counterfactual experiments imply that operational knowledge accounts for about 1/3 of VCs' impact on aggregate growth. Policy experiments suggest that changes affecting the VC market could result in a 7 basis point gain in the long-run growth rate of the economy.

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“... We were cash positive. We didn’t have a year where we lost money... We eventually ... sold 5% of the company for a million dollars ... just to get a venture capital company to join our board and give us some adult advice... That money sat in the bank.”

Bill Gates, ex-chairman and co-founder of Microsoft

## 1 Introduction

Investments by venture capital (VC) companies have a disproportionate impact on the US economy. In 2010, the revenues of firms that had ever received VC support accounted for 21% of GDP, and their employment share was 11% of total private sector employment, although VC investments to their portfolio companies amount to less than 0.2% of GDP.<sup>1</sup> VC financing is of particular relevance for firm creation and innovation because VC firms strive to find young and innovative firms that lack market experience. VC firms are unique in that they do not only provide financing: They also actively engage in management by bringing their *operational knowledge* to bear in their investments.<sup>2</sup> Despite this distinctive structure, the contribution of operational knowledge to firm productivity and its implications for aggregate economic growth lack a thorough investigation. This paper presents a rigorous quantitative framework to explore the distinct mechanisms by which VC firms influence innovative firms and, through them, aggregate productivity.

Investing in a young company that needs to develop an innovative business idea entails considerable uncertainty and is subject to pervasive moral hazard problems (Gompers, 1995). In such environments, Casamatta (2003) shows that the *optimal contract* specifies a “dual role” for the VC firm. The optimal contract bundles financing and advice so that a VC firm’s financial stake in the company motivates it to provide valuable advice. By contrast, consulting firms are not preferred by young and innovative firms because they do not acquire stakes in the latter.<sup>3</sup> As a consequence of this lack of “skin in the game,” entrepreneurs have to pay a very high price in order to obtain valuable advice from the consultant. Therefore, young and innovative companies prefer VC advising to consulting advice.

Taking the structure of the optimal contract as given, I develop a structural model containing an explicit VC market. This model serves two main purposes: Firstly, I use the model to measure

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<sup>1</sup>National Venture Capital Association (NVCA, 2013). Well-known examples include Amazon, Google, Starbucks, and FedEx.

<sup>2</sup>By operational knowledge, I refer to a general body of expertises concerning organizational structure, marketing, product development, and other business domains. This role of venture capital financing reflects the idea “that the typical founder is an incomplete businessman, with gaps in experience in matters such as financial management and marketing. An active board of directors, staffed by representatives of the [venture capital] investors, is expected to help fill these gaps” (Bartlett, 1995). For evidence on different methods, see Gorman and Sahlman (1989), Sahlman (1990), Gompers (1995), and Hellmann and Puri (2000), among others. Da Rin et al. (2011) provide an extensive survey.

<sup>3</sup>The result assumes that the entrepreneurial effort is cheaper and is key for the success of the project.

the importance of the VCs' *operational expertise* to firm growth. To identify this channel, it is fundamental to separate it from the provision of financing and the overall selection of "portfolio" firms by VCs. Establishing a unified structure that accounts explicitly for different aspects of the VC market, the structural model is an effective tool to accomplish this task. Quantifying the operational knowledge channel is useful for evaluating the advantages commonly attributed to VC finance in fostering firm productivity and growth. To the extent that VC companies add valuable knowledge to their undertakings, they become a more efficient option for financing innovation than more traditional financing sources such as bank loans. Secondly, the model provides a suitable ground to shed light on various policy discussions such as the relationship between an active public equity market and VC financing.

To address these issues, I propose a new dynamic general equilibrium model of innovation with heterogeneous firms, in the tradition of [Romer \(1990\)](#), [Grossman and Helpman \(1991\)](#), and [Acemoglu et al. \(2013\)](#), among others.<sup>4</sup> In this model, entrepreneurs/private firms produce differentiated goods of heterogeneous quality which they can improve through risky research and development. The efficiency of this development process can only be increased with the help of a VC and is otherwise fixed. I introduce to this model a detailed venture capital market through (i) the feature of VC expertise and (ii) the endogenous matching market for firms and VCs. Every private firm that is not in a relationship with a VC can search for VCs and meet them in the matching market. VCs improve the efficiency of product development through their operational knowledge. They also provide financing and relax the cost of inputs into the production of goods. This financial support to a priori unconstrained firms reflects the dual structure of the optimal contract for VC investment. The heterogeneity in the quality level of private firms determines the magnitude of the improvement that VC firms can potentially create. VC firms are subject to entry costs, which induce them to select firms for investment that present more room for growth. Thus, in addition to financing and operational knowledge channels, the VC setting also accounts for the effects of selection by VC firms. Because the preferred option for VC firms to exit their portfolio companies is to sell them via initial public offerings, the model also includes a public equity market. To complete the general equilibrium framework, the rest of the structure builds on the shoulders of endogenous technical change models in which entrepreneurs own intermediate product lines. Entrepreneurs enter the market with a new product line while intermediate good producers who cannot develop the quality of their good sufficiently exit the economy. Together with entry and exit margins, the innovations generated by these intermediate good firms determine the endogenous rate of growth of the aggregate economy.

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<sup>4</sup>The model shares features such as product variety expansion with [Romer \(1990\)](#), quality ladder structure with [Grossman and Helpman \(1991\)](#), and innovation by incumbent firms with [Acemoglu et al. \(2013\)](#), whose details are explained below.

The main identification problem in this model is to distinguish the financing channel from operational knowledge in their relative influence on firms receiving VC investment. The assumption that disentangles these two channels is that the former mainly affects the level of the profits while the latter changes its growth rate. The financial help of VC decreases the cost of inputs in the intermediate good production. Therefore, the entrepreneur earns higher profits for a given quality level of the intermediate good that the firm produces. Operational knowledge, on the other hand, directly affects the efficiency of the entrepreneur's effort in generating innovations that increase the product quality.<sup>5</sup> To discipline the size of the financial impact in the model, I target the ratio of VC investment to GDP. Determining the size of VCs' contribution to efficiency in the data is a more delicate task. A well-known concern is selection: VCs might be "cherry-picking" already good firms instead of improving them in some other way. I address this issue by applying the method of indirect inference in my estimation. To do so, I utilize the findings of [Puri and Zarutskie \(2012\)](#). In their empirical study, [Puri and Zarutskie \(2012\)](#) provide statistics on growth rates of VC-backed and non-VC-backed private firms, controlling for selection on observable characteristics. In a nutshell, [Puri and Zarutskie \(2012\)](#) create samples of private firms with and without VC support that are matched on some measurable features. Following similar steps, I create the analogues of such samples from the stationary firm distribution of my model. Finally, I use the model-generated samples to match the regression statistics on VCs' effect on firm growth provided in the same paper.

I estimate this model by the method of moments, using US data on the venture capital market, public equity issuances, and research and development expenditures. The model does a successful job in matching moments that pertain to venture capital and innovation aspects of the model, such as the duration of VC matches, firm age at the issuance of initial public equity, and aggregate share of R&D. Moreover, the model-generated regression results accurately predict the coefficient estimate found in [Puri and Zarutskie \(2012\)](#). Successfully hitting this target via indirect inference is crucial because it determines the scope of the influence of the operational knowledge channel. Before using the estimated model for counterfactual analyses, I compare its auxiliary predictions to data moments in order to obtain out-of-sample validation. This comparison reveals that the model is very precise in capturing the high IPO frequency among VC-backed firms and the share of IPOs issued by VC-backed companies, both of which are definitive characteristics of the VC market in the US.

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<sup>5</sup>[Gonzalez-Urbe \(2014\)](#) is a recent empirical work that points towards the efficiency enhancing role of VC. By using the introduction of the Prudent Investor Rule (PIR) across states as a source of exogenous variation in VC financing, she first documents a 50% rise in the annual citations of patents of a firm after it obtains VC financing. More interestingly, she shows that the probability of receiving a citation from a company in the portfolio of the same VC firm increases twice as much as from a company outside the VC's portfolio. This result indicates that VCs facilitate the diffusion of knowledge among their portfolio companies. Similarly, [Lindsey \(2008\)](#) argues that by mitigating informational and contractual problems, VC firms increase the probability of strategic alliances among their portfolio companies. The empirical estimates imply a 70% increase in the probability of R&D alliances, a significant constituent of strategic cooperations.

The first set of counterfactual experiments determines the relevance of VCs' operational knowledge to firm and aggregate growth. I create hypothetical economies in which I strengthen particular channels of VC finance in each experiment. Comparing the responses of the aggregate growth rate to these changes demonstrates that the knowledge channel accounts for 1/3 of VCs' impact. Hence, the conclusion is that VC support matters significantly beyond financing. Then I consider a 15% increase in the fixed cost of IPO to capture the average level of underwriting spreads in the US before their secular decline after the 1980s.<sup>6</sup> As a result, fewer private firms issue IPOs, and the equilibrium probability of a match with a VC firm decreases. Thus, the increase in the fixed IPO cost results in a smaller share of VC-backed private firms in the economy. This leads to a 1.5 basis point loss in the long-run growth rate.

As an example for the policy implications of the model, I consider a recent regulation that the European Union introduced in 2013 regarding European VC firms. In order to decrease the fundraising costs of VC firms, this policy aims to harmonize the legislative environment these firms face when investing across the borders of European countries. I map this change into the model as lower entry costs for VC firms through lump-sum subsidies. I find that this policy can increase the long-run growth rate by 7 basis points at a cost of subsidies that corresponds to approximately 8% of the VC investment in the model. This increase in the growth rate hinges on the reallocation of private firms towards the VC market. Moreover, a rise in the median duration of the VC-firm relationship amplifies the effect of the operational knowledge. These results highlight the significance of the general equilibrium effects for the policy evaluation.

**Related Literature.** This paper draws on several strands of the literature. First, by embedding the VC market into the endogenous technical change environment, it contributes to the literature that concentrates on innovation and firm dynamics (Klette and Kortum, 2004; Akcigit, 2010; Akcigit and Kerr, 2017; Lentz and Mortensen, 2008; Acemoglu et al., 2013).<sup>7</sup> Lentz and Mortensen (2008) and Acemoglu et al. (2013) are recent examples that particularly focus on allocation of resources across firms with heterogeneous capacities to innovate. This paper contributes to the analysis by introducing a link between this heterogeneity and the financing decisions of innovative firms. In that regard, this paper also relates to work on finance of innovation (Aghion and Tirole, 1994; Aghion et al., 2004; Kortum and Lerner, 2000; Lerner et al., 2011; Brown et al., 2009; Amore et al., 2013).<sup>8</sup> As an example in the setting of endogenous growth, Itenberg (2014) explores the effect of developments in the US public equity market on R&D decisions of small firms. By contrast, the

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<sup>6</sup>An underwriting spread, also known as gross spread, measures the fees paid to the underwriter of the issue in compensation for expenses such as legal expenses, management fees, etc. as a fraction of the total proceeds raised. This spread is a direct cost associated with the issue, which I model as a fixed cost.

<sup>7</sup>For a detailed discussion of innovation and firm dynamics in the context of Schumpeterian growth theory, see Aghion et al. (2013).

<sup>8</sup>Hall and Lerner (2009) is a seminal survey on this topic.

focus of this paper is the venture capital market.

This study extends the venture capital literature by analyzing VC financing in a dynamic quantitative framework. The theoretical work in this area uncovers the conditions in a static setting that leaves room for the use of venture capital in the existence of alternative financing or in advising agents such as banks and consultants (Amit et al., 1998; Casamatta, 2003; Ueda, 2004; de Bettignies and Brander, 2007).<sup>9</sup> While my work acknowledges these theoretical foundations, and borrows the features of the optimal VC contract from this literature, it focuses on quantitative analysis of the VC market. In particular, my model improves the understanding of VC financing in a dynamic general equilibrium setting that enables the measurement of the distinct channels through which VC firms affect firm dynamics. A realistic structure for the VC market in a model of endogenous firm dynamics also allows the analysis of venture capital from the perspective of innovation policy. Due to these characteristics, this work also contributes to answering empirical questions in VC literature.<sup>10</sup> In particular, Kortum and Lerner (2000) show the significant effect of VC finance on firm-level innovation in terms of both patent counts and citations, whereas papers such as Hellmann and Puri (2000, 2002) examine the effectiveness of particular management practices applied by VC firms using hand-collected data. My paper advances these exercises to quantify VCs' impact by using a new structural model as a measurement tool that takes into account important margins such as selection and reallocation. Furthermore, the setting should also be helpful in shedding light on various policy debates, such as the relationship between public equity and VC markets (Black and Gilson, 1998; Bottazzi and Rin, 2002).

Finally, a related literature focuses on the role of the financial system in evaluating and selecting investment projects. For instance, Jayaratne and Strahan (1996) show empirically that interstate branch reform in the US banking system has led to a tighter selection in lending through increased competition. This in turn has resulted in higher lending quality and growth rates in liberalized states. In the context of VC, Casamatta and Haritchabalet (2007) provide evidence on how VC firms use syndication practices to obtain a second opinion when deciding on early rounds of investment. By estimating a theoretical framework using Bayesian methods, Sørensen (2007) finds important effects of assortative matching in the VC market. In relation to this literature, my paper formalizes the idea that VC firms search for high growth potential by embedding an endogenous search and matching market that accounts for selection. This aspect is integral to identifying the effect of "value-adding" practices of VC firms in the model. Furthermore, building

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<sup>9</sup>There are a few recent papers that include venture capital in a dynamic setting. Silveira and Wright (2013) model VC firms in a search and matching environment and analyze its theoretical predictions for the life cycle of VC firms. Pinheiro (2012) examines the theoretical underpinnings of the optimal duration in matches between VC firms and their undertakings. Opp (2014) analyzes the cycles in the venture capital market. None of these studies consider the effects on firm-level innovation and economic growth.

<sup>10</sup>For a recent survey, see Da Rin et al. (2011).

on the endogenous growth framework, this paper relates to the literature that analyzes the effects of selection on economic growth (King and Levine, 1993; Jaimovich and Rebelo, 2017).<sup>11</sup> A recent paper in this strand, Ateş and Saffie (2016), argue that financial selection introduces a trade-off between the mass and the quality in firm entry, and analyzes its impact on aggregate productivity growth in the context of sudden stops.<sup>12</sup> In comparison to this literature, my work focuses on a specific financial intermediary, but one that is more relevant to innovation and growth, namely venture capital finance.

The next section introduces the main ingredients of my model. Section 3 explains the data used in the estimation, and discusses identification. Section 4 presents counterfactual and policy experiments, and Section 5 concludes.

## 2 Model

In this section, I present the components of the model economy. Entrepreneurs produce differentiated intermediate goods and sell them to a representative final good producer that combines these intermediate goods into a final output. The entrepreneurs enter the business with an innovative product, the quality of which they can improve over time by investing in risky research and development (R&D) activities. The efficiency of the R&D process is a firm characteristic that is fixed unless the firm uses the additional business expertise of a VC. VC support also entails a reduction in the cost of intermediate good production. An intermediate good producer can search for and match with a VC firm in the endogenous search and matching market. The contribution of the VC to firm growth stems from increased R&D efficiency that makes product developments likelier. However, it comes at the cost of firm dilution, and carries an exogenous risk of running the project idle. Any private firm can issue public equity to expand the size of its enterprise, but there is an associated fixed cost. By improving the efficiency of product development, VC firms help their undertakings raise adequate resources faster to afford the IPO cost.

### 2.1 Preferences

Consider the following closed economy in discrete time. The representative household maximizes the expected discounted sum of the period utility from consumption with the following preferences

$$U_t = \sum_{\tau=t}^{\infty} \beta^{\tau} \frac{C_{\tau}^{1-\varepsilon} - 1}{1-\varepsilon}$$

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<sup>11</sup>Levine (2005) provides an excellent survey on this topic, and on the relationship between finance and growth, in general.

<sup>12</sup>In a related paper, Ateş and Saffie (2013) provide a theoretical characterization of the nexus between financial selection and the long-run productivity growth by analyzing the mass-quality trade-off in a closed economy setting.

where  $C_t$  denotes consumption at time  $t$ .  $\beta$  is the discount factor and  $\varepsilon$  is the coefficient of relative risk aversion. The household consumes a final good, and supplies labor inelastically to the final good producer, which I normalize to 1 without loss of generality. Households own all the firms in the economy, and their budget constraint is

$$C_t \leq \int_{j \in J} \Pi_{jt} dj + w_t \quad (1)$$

where  $\Pi_{jt}$  is the flow profit of the intermediate firm  $j$  in the interval  $J$  of actively operating firms, and  $w_t$  is the wage level at time  $t$ .

## 2.2 Final Good Production

The final good, which is used for consumption, R&D, and intermediate good production, is produced in a perfectly competitive market. The production technology combines labor and differentiated intermediate varieties in the following structure:

$$Y_t = \frac{1}{1-\alpha} L_t^\alpha \int_{j \in J} q_{jt}^\alpha k_{jt}^{1-\alpha} dj. \quad (2)$$

Here,  $L_t$  denotes the labor input,  $k_{jt}$  refers to intermediate good  $j \in J$  at time  $t$ , and  $q_{jt}$  is the associated quality of product  $j$ .  $(1-\alpha)$  stands for the physical factor share.  $Y_t$  is the numeraire good in the economy.

The representative final good producer chooses a bundle of intermediate goods and labor in order to maximize its profits. Taking the price of the intermediate product,  $p_{jt}$ , as given, the problem of the final good producer reads as:

$$\Pi_{Y,t} = \max_{L_t, \{k_{jt}\}_{j \in J}} \left\{ \frac{1}{1-\alpha} L_t^\alpha \int_{j \in J} q_{jt}^\alpha k_{jt}^{1-\alpha} dj - \int_{j \in J} p_{jt} k_{jt} dj - w_t L_t \right\}. \quad (3)$$

The solution of this maximization problem yields in equilibrium the following inverse demand for intermediate good  $j$ :

$$p_{jt} = q_{jt}^\alpha \left( k_{jt}^d \right)^{-\alpha} \quad (4)$$

where  $k_{jt}^d$  is the optimal amount of good  $j$  demanded by the final good producer.

## 2.3 Intermediate Good Firms

Intermediate firms are distributed across product lines whose measure,  $J_t$ , is determined endogenously. There are three types of intermediate good firms: private firms that are not matched with a VC, private firms that are matched with a VC, and public firms. Each firm is characterized mainly

by two state variables which are the product quality and the R&D efficiency. An entrepreneur that has an innovative project enters a product line as a private firm without a VC. First I introduce decisions that are common to all intermediate good firms, and then I go into the specific choices of different types of firms.

### 2.3.1 Production

Each intermediate good is a monopolist in producing its differentiated good  $k_{jt}$ . To maximize the operating profits, the monopolist solves the following problem

$$\begin{aligned} \Pi_{jt}(q_{jt}) &= \max_{k_{jt}} \{p_{jt}k_{jt} - C_k(k_{jt})\} \quad \text{subject to} \\ p_{jt} &= q_{jt}^\alpha (k_{jt}^d)^{-\alpha} \end{aligned}$$

where  $C_k(k_{jt})$  denotes the cost of inputs to produce  $k_{jt}$  amount of intermediate good in terms of the final good and has the following form:

$$C_k(k_{jt}) = \eta_j k_{jt}.$$

In this specification,  $\eta_j \in \{\eta^H, \eta\}$  denotes the marginal cost of production with  $\eta^H > \eta$ . For any firm that does not have VC support, this parameter has the higher value. Therefore, this structure captures the financial contribution of VC firms to their undertakings. An interpretation for this structure is that it reflects cash-in-hand constraints in a reduced form way. In line with reality, VC relaxes this financial constraint with its monetary commitment.

In equilibrium, the optimal level of intermediate good production becomes

$$k_{jt} = \alpha \left[ \frac{1 - \alpha}{\eta_j} \right]^{\frac{1-\alpha}{\alpha}} q_{jt}. \quad (5)$$

With a constant mark-up over price, this optimal quantity generates profits that are linear in product quality  $q_{jt}$ . Thus,  $\Pi_{jt} = \pi_j q_{jt}$  where

$$\pi_j = \begin{cases} \pi^L & \text{if } C_k(k_{jt}) = \eta^H k_{jt} \\ \pi^H & \text{if } C_k(k_{jt}) = \eta^L k_{jt} \end{cases}$$

is a constant depending on the marginal cost  $\eta_j$ . Hence, the operational profits are higher if the marginal cost of intermediate good production is lower due to VC support.

### 2.3.2 Research and Development

Each firm invests in R&D to improve the quality of its product and hence to increase its operating profits. Let  $i_{jt}$  and  $Z(i_{jt})$  denote the (process) innovation rate and the R&D effort required to generate this rate, respectively. The R&D cost function in terms of the final good has the following form:

$$Z(i_{jt}) = \frac{h(i_{jt})}{\theta_j} q_{jt}$$

where  $h(\cdot)$  is a convex, strictly increasing function. In this specification,  $\theta_j$  denotes the efficiency in developing the product quality.

In order to analyze the effect of VC firms' operational knowledge, the parameter  $\theta$  can take three different values:

$$\theta_j = \begin{cases} \theta^L & \text{for private firms without VC} \\ \theta^H & \text{for private firms with VC} \\ \theta_j^{pb} \in \{\theta^L, \theta^M\} & \text{for public firms} \end{cases}$$

In this economy, the private firms conduct R&D with low efficiency,  $\theta^L$ , unless they receive help from a VC firm. Once matched, a VC firm raises the efficiency level of its portfolio company to  $\theta^H$  thanks to its expertise. In turn, a higher efficiency in generating innovations increases the expected growth rate of the private firm. In addition to this direct effect, I allow for the possibility that VC firms also cause a permanent effect. Product development efficiency after IPO,  $\theta_j^{pb}$ , depends on whether the firm used VC finance or not. The underlying motivation is that, although the entrepreneur separates from the VC, she retains some of the operational skills brought to the firm by the venture capitalist. Therefore, for a firm that becomes public under the supervision of a VC, I assume that  $\theta_j^{pb} = \theta^M$ . Although determined by the data through estimation, it is expected that  $\theta^H > \theta^M > \theta^L$ , as validated in Section 3.4. The ordering  $\theta^H > \theta^M$  reflects the loss of VC supervision, whereas  $\theta^M > \theta^L$  reflects the VC's permanent impact on the firm's operational knowledge stock. The product development efficiency remains constant unless the firm changes its type due to a financial decision.

A successful process innovation improves the product quality of the firm by an amount that is taken to be proportional to the average quality of the firm,  $\bar{q}_t$ :

$$q_{jt+1} = q_{jt} + \lambda \bar{q}_t.$$

where  $\bar{q}_t \equiv \int_{j \in J} q_{jt} dj$ . If the R&D is unsuccessful,  $q_{jt}$  remains the same. These additive increments in product quality introduce decreasing returns to innovation, and imply smaller incentives to

innovate for larger firms. Limiting the growth potential of larger firms, this structure enables the model to generate a stationary size distribution in equilibrium.

### 2.3.3 Free Exit

Every intermediate firm has an outside option  $\chi_t^o = \chi^o \bar{q}_t$  which is proportional to the average quality  $\bar{q}_t$ . If the value of the firm goes below this level, the firm exits the economy. Notice that the option value grows at the rate of the aggregate economy. Therefore, if a firm fails to innovate for a long period of time, it will necessarily exit the market. Therefore, in addition to the profit enhancing motive, there is another incentive to motivate, namely to survive in the business.

### 2.3.4 Free Entry

The economy has a unit measure of potential entrants. These outside firms need to generate an innovation to enter the market. An entering firm observes the initial quality of its product upon successful innovation. This initial quality is drawn from the stationary distribution of the previous period, but from a range that is small enough such that the entrant does not go public immediately.<sup>13</sup> An entrant opens a new intermediate product line and starts with the low level of product development efficiency  $\theta^L$ .<sup>14</sup>

The cost of generating a product innovation for entry is quadratic in probability of innovation,  $x_t$ :

$$C_e(x) = \chi^e \bar{q}_t f(J_{t-1}) x_t^2$$

where  $\chi_e$  denotes the scale of the cost function. There are two important features in this cost structure. First, it is proportional to aggregate productivity level  $\bar{q}_t$ . Since the expected value of an innovation also shares this proportionality the optimal innovation rate becomes independent of  $\bar{q}_t$ . Second, the cost depends on the previous measure of the intermediate firms through a convex and increasing function.<sup>15</sup> This structure relates the measure of firms to the size of entry, and enables the economy to reach a stable size.

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<sup>13</sup>Given the median age of US firms at the IPO stage, this assumption is a plausible one. Moreover, the average size of entry firms in the US are drastically smaller than the average size of incumbent firms (Scarpetta et al., 2004; Bartelsman et al., 2009).

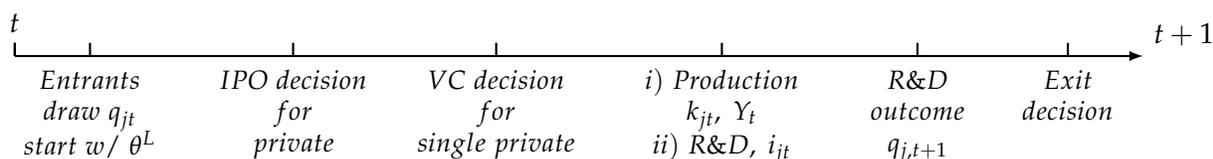
<sup>14</sup>The fact that entrants open new product lines introduces a source for growth due to expanding product markets, à la Romer (1990). However, as explained below, the measure of intermediate product lines,  $J$ , remains constant in a balanced growth path equilibrium.

<sup>15</sup>In the estimation,  $f(\cdot)$  is assumed to have a quadratic form. This type of relationship has the interpretation that the resources to innovate are scarce, and the costlier it is for entrants to use these resources, the larger the share of incumbent firms becomes.

### 2.3.5 Timing of Events

The timing of events is summarized in Figure 1. The period starts with the entrants' decision to pay the entry fee and draw their productivity. Then, the private intermediate good producers make their financial decisions, which are searching for VC and going public. Then, the final good producer and intermediate good producers decide on production. Intermediate good producers also determine their innovation intensities. Lastly, the R&D outcomes are realized, and intermediate producers make their exit decisions.

Figure 1: Flow of Events



Next, I explain the different types of intermediate firms and their specific financial choices. In particular, a public firm will consider only the decisions introduced above. Every private firm considers going public at the onset of a period, in addition to the aforementioned common decisions. Lastly, a private firm that is not matched with a VC can search for a VC if it has already chosen to remain private.

### 2.3.6 Firm Types and Financial Decisions

**Public Firm.** A private firm can choose to go public by issuing an initial public offering (IPO) and raise public funds to expand its operations.<sup>16</sup> I assume that a firm cannot look for a VC and cannot raise any public funds once it is public. Therefore, the only decisions that a public firm needs to consider are production, R&D, and exit decisions. Specifically, let  $V_t^{pb}$  denote the value of a public company. Then the problem of the public firm becomes

$$V_t^{pb}(q_{jt}, \theta_j^{pb}) = \max_{i_{jt}} \left\{ \begin{array}{l} \pi^L q_{jt} - h(i_{jt}) / \theta_j^{pb} \cdot q_{jt} + \frac{1}{1+r_{t+1}} \times \\ [i_{jt} W_t^{pb}(q_{jt} + \lambda \bar{q}_t, \theta_j^{pb}) + (1 - i_{jt}) W_t^{pb}(q_{jt}, \theta_j^{pb})] \end{array} \right\} \quad (6)$$

where  $r_{t+1}$  denotes the interest rate. The continuation value is defined as

$$W_t^{pb}(q_{jt+1}, \theta_j) = \max \left\{ \chi^o \bar{q}_t, \mathbb{E} V_{t+1}^{pb}(q_{jt+1}, \theta_j^{pb}) \right\}. \quad (7)$$

<sup>16</sup>This type of modelling IPO is in line with the investment financing explanation of equity finance. Using extensive data on initial and seasoned equity offerings across 38 countries during the period 1990-2003, Kim and Weisbach (2008) show that firms subsequently use proceeds from selling equity for R&D and CAPEX investments. When I explain the decision to do an IPO below, it will be clear that going public also provides an exit channel for the VC.

Every period, the public firm collects flow profits and decides on the optimal size of process innovation.<sup>17</sup> In case of successful R&D, it increases its product quality with which it starts the next period, unless it chooses to use the outside option and exits. If R&D efforts do not result in an incremental innovation, the product quality remains the same. In this regard, product development enables the firm to decrease the likelihood of exiting the market, besides increasing the profits. Note that the product development efficiency remains constant in any case.

**VC decision.** A private firm without VC backing can search for a VC in every time period. To understand the benefit of becoming matched with a VC, first consider the problem of a private firm with a VC that decided not to go public. This post-IPO-decision value is the one that a private firm without VC obtains when it is matched with a VC firm, because the process of searching for a VC follows the IPO decision. It is defined as

$$V_t^{pr} (q_{jt}, \theta^H, \mathbb{I}_j^{vc} = 1) = \max_{i_{jt}} \left\{ \begin{array}{l} \pi q_{jt} - h(i_{jt}) / \theta^H \cdot q_{jt} + \frac{1}{1+r_{t+1}} \times \\ \left[ \begin{array}{l} i_{jt} W_t^{pr} (q_{jt} + \lambda \bar{q}_t, \theta^H, 1) + \\ (1 - i_{jt}) W_t^{pr} (q_{jt}, \theta^H, 1) \end{array} \right] \end{array} \right\}. \quad (8)$$

where  $\mathbb{I}_j^{vc}$  is an indicator function that takes the value 1 if the firm is matched to a VC. This problem is very similar to the one of the public firm, with three exceptions. First, as the firm gets the expertise of VC, its efficiency increases to level  $\theta^H > \theta^L$ , and this is the *operational knowledge* channel through which a VC adds value to the firm. Second, due to its financial support, VC increases the profits of the firm to  $\pi > \pi^L$  for a given quality level. The third difference reflects the fact that many relationships between the VC and the firm end up unsuccessfully, i.e. they do not lead to any IPO or acquisition by another firm where VC can have a profitable exit. To capture this, I assume that the VC and the firm can separate with an exogenous probability  $\sigma^{vc}$  in which case the firm exits the market. The continuation value  $W_t^{pr}(\cdot, 1)$  incorporates these differences.

A private firm meets with VCs in a random matching environment.<sup>18</sup> The endogenous probability that a private firm matches with a VC firm is defined by

$$m_f(\Lambda) = \rho \frac{\Lambda}{1 + \Lambda}$$

where  $n_{vc}$  and  $n_f$  denote the number of VC firms and available private firms, respectively,  $\Lambda \equiv n_{vc}/n_f$  is the market tightness, and  $\rho$  refers to the efficiency parameter (Shi, 2009).<sup>19</sup> Then the

<sup>17</sup>Notice the low level of profits because public firms do not have VC support.

<sup>18</sup>The main friction in this market is the process of evaluation by the VC. It is a cumbersome process in which only one out of a hundred applicants gets funded on average, according to NVCA figures. A directed search on the firm side is also unlikely given the low probability of acceptance. Moreover, this would require the applicant to gain information about other companies in the portfolio the VC firm of interest, their financing stages, the human capital constraints of the VC firm, etc., which is probably not the case with most of the applications in reality.

<sup>19</sup>Correspondingly, the total number of matches is given by the so-called telegraph matching function.

value of a private firm without a VC, after deciding not to issue an IPO, becomes:<sup>20</sup>

$$\begin{aligned}
 V_t^{pr} (q_{jt}, \theta^L, \mathbb{I}_j^{vc} = 0) &= m_f \left( \frac{n_{vc}}{n_f} \right) \mathbb{I}_{s^{vc} > 0} \times V_t^{pr} (q_{jt}, \theta^H, 1) + \\
 &\left[ \left( 1 - m_f \left( \frac{n_{vc}}{n_f} \right) \right) + m_f \left( \frac{n_{vc}}{n_f} \right) (1 - \mathbb{I}_{s^{vc} > 0}) \right] \times \\
 &\max_{i_{jt}} \left\{ \begin{array}{l} \pi^L q_{jt} - h(i_{jt}) / \theta^L \cdot q_{jt} + \frac{1}{1+r_{t+1}} \times \\ \left[ i_{jt} W_t^{pr} (q_{jt} + \lambda \bar{q}_t, \theta^L, 0) + (1 - i_{jt}) W_t^{pr} (q_{jt}, \theta^L, 0) \right] \end{array} \right\}. \quad (9)
 \end{aligned}$$

Here,  $m_f(\cdot)$  denotes the probability that the firm will meet with a VC. The share of the firm to be left to VC,  $s^{vc}$ , is determined endogenously via Nash bargaining. The first line of this value simply tells that, if there is a match that generates a positive surplus the firm, matches with a VC. Otherwise, it moves on to make production, R&D and exit decisions where its R&D efficiency remains constant.

**Private firms and IPO decision.** Any private firm, with or without a VC, can issue an IPO in any period. The upside of IPO is an increase in the size of operations. Moreover, it enables the VC firm sell its share in the company and collect the return.<sup>21</sup> Let  $\mathcal{V}_t^{pr}$  refer to the value of a private firm that considers going public or remaining private. Then the IPO decision is determined by the following maximization

$$\mathcal{V}_t^{pr} (q_{jt}, \theta_j, \mathbb{I}_j^{vc}) = \max \left\{ V_t^{pr} (q_{jt}, \theta_j, \mathbb{I}_j^{vc}), (1 - \Delta) V_t^{pb} (\kappa q_{jt}, \theta_j^{pb}) - \chi^{ipo} \bar{q}_t \right\}.$$

The first part of this maximization is the value of the firm if it remains private.  $V_t^{pb} (\kappa q_{jt}, \theta_j^{pb})$  in the second part denotes the value of the public firm with larger size of operations, where  $\kappa > 1$  denotes the increase. Firms that issue an IPO without any previous relationship with a VC do not experience any change in the efficiency of product development.

At last, the firm incurs various costs of issuing an IPO, which are captured by  $\chi^{ipo} \bar{q}_t$ . Moreover, a  $\Delta$  share of the firm value is sold at IPO. The firm finances its investment in improving its product quality with the proceeds from this transaction. In addition, if the firm that goes public is matched with a VC, IPO allows the VC to liquidate its stocks in the company in order to obtain the return on its initial investment,  $s_j^{vc}$ .

<sup>20</sup>No search costs are assumed in this setting.

<sup>21</sup>IPO is considered as the most profitable exit option and a measure of success for VC funds (Brander et al., 2002; Sørensen, 2007). According to the 2013 Yearbook of National Venture Capital Association, 16% of portfolio companies end up going public.

## 2.4 Venture Capital Firms

Venture capital firms are agents that provide operational knowledge and finance to private firms. There is an outside pool of VC firms. To enter the matching market, a VC firm has to pay an entry cost. The entry cost is given by  $\lambda_t^{vc} \equiv \chi^{vc} \bar{q}_t$ , and is proportional to the average productivity in the economy. At any point in time, a VC firm can be matched with only one firm. When the VC exits its investment, it is assumed to exit the economy. The value of a VC that is not matched to a private firm is

$$A_t = m_{vc} \left( \frac{n_{vc}}{n_f} \right) \int s_t^{vc} (q, \theta^L; \cdot) \Psi_q (dq) + \left[ 1 - m_{vc} \left( \frac{n_{vc}}{n_f} \right) \right] \frac{A_{t+1}}{1 + r_{t+1}} \quad (10)$$

where  $\Psi_q$  denotes the distributions over  $q$  of the private firms that are in the matching market. The first part of equation (10) explains that, with probability  $m_{vc}(\cdot)$ , the VC meets a private firm and gets a share  $s^{vc}$ . Otherwise it continues to search next period.

The share of the firm that the VC gains,  $s_t^{vc}(q, \theta)$ , is the solution of the following Nash bargaining problem

$$s_t^{vc}(q, \theta) = \arg \max_s \left[ V_t^{pr}(q, \theta^H, 1) - V_t^{pr}(q, \theta^L, 0) - s \right]^{1-\phi} \left[ s - \frac{A_{t+1}(\cdot)}{1 + r_{t+1}} \right]^\phi \quad (11)$$

where  $\phi$  is the bargaining power of the VC. Notice that, for a match to form between a VC firm and a private company, the payment to the VC firm needs to be a positive amount because the VC firm is subject to an entry cost. This implies that the VC firms invest in companies only if the expected surplus is larger than zero. This selection margin is integral for the identification purposes.<sup>22</sup>

## 2.5 Equilibrium

Throughout this paper, I will focus on the Markov Perfect Equilibrium. In particular, the analysis will be based on the balanced growth equilibrium where aggregate variables grow at a constant rate. To this end, it will be necessary to transform the economy into a stationary one by normalizing the growing variables by the aggregate productivity  $\bar{q}_t$ . First, I denote  $\hat{q}_{jt} \equiv q_{jt} / \bar{q}_t$  as the normalized quality. Next, I define the Markov Perfect Equilibrium where the asterisk refers to equilibrium values.

**Definition 1 (Equilibrium)** Let  $\zeta_j^d \in \{0, 1\}$ ,  $d \in \{exit, vc, ipo\}$ , denote the decisions of firm  $j$  regarding exit, VC search, and going public, respectively. A Markov Perfect Equilibrium consists of aggregate prices  $\{r_t^*, w_t^*\}$ ; aggregate output, consumption, R&D expenditure, and intermediate input expenditure

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<sup>22</sup>See Section 3.2 for details.

$\{Y_t^*, C_t^*, Z_t^*, K_t^*\}$ ; intermediate prices and quantities  $\{k_{jt}^*, p_{jt}^*\}$ ; R&D, exit, search, and floating decisions  $\{i_{jt}^*, \xi_j^{d*}\}_{d \in \{\text{exit}, \text{vc}, \text{ipo}\}}$ ; firm value functions  $\{V_t^{f*}, W_t^{f*}\}_{f \in \{\text{pb}, \text{pr}\}}$  and VC value  $A_t^*$ ; VC pricing function  $\mathbf{s}_t^{\text{vc}*}$ ; the normalized quality distribution and the mass of firms  $\{\hat{\Psi}_t^*(\hat{q}), J_t^*\}$  where  $t \in [0, \infty)$ ,  $j \in J_t^*$  such that

1.  $\{k_{jt}^*, p_{jt}^*\}$  are given by (5) and (4), and maximize the operating profits,
2.  $\{V_t^{f*}, W_t^{f*}\}_{f \in \{\text{pb}, \text{pr}\}}$  satisfy (6), (7), (9), and (8),
3.  $i_{jt}^*$  maximize the expected profits, and  $\{\xi_j^{d*}\}_{d \in \{\text{exit}, \text{vc}, \text{ipo}\}}$  solves the value functions,
4.  $\hat{\Psi}_t^*(\hat{q})$  is consistent with R&D, entry, exit, VC, and IPO decisions of the firms,
5.  $J_t^*$  supports the free entry condition to hold with equality,
6.  $A_t^*$  is given by (10),
7.  $\mathbf{s}_t^{\text{vc}*}$  as in (11) is determined by Nash bargaining;
8.  $\{Y_t^*, C_t^*\}$  are given by (2) and (1),
9. and aggregate prices  $\{r_t^*, w_t^*\}$  clear the market.

Accordingly, a balanced growth equilibrium is defined as follows:

**Definition 2 (Balanced Growth Path)** A Balanced Growth Path (BGP) is an equilibrium where  $\hat{\Psi}^*(\hat{q})$  defines an invariant distribution, the measure of firms,  $J^*$ , has a fixed value, and the average quality  $\bar{q}$  and the aggregate variables grow at a constant rate  $\mathbf{g}$ .

Given the invariant distribution of normalized quality levels and the stationary R&D decisions, I can now derive the constant growth rate of the economy in a BGP:<sup>23</sup>

$$\begin{aligned}
 \mathbf{g} &= \int_{j \in J^*} \left(1 - \xi_j^{\text{exit}}\right) \left\{i_j \lambda + \xi_j^{\text{ipo}} (\kappa - 1) (\hat{q}_j + i_j \lambda)\right\} dj \\
 &\quad - \int_{j \in J^*} \left\{\xi_j^{\text{exit}} + \mathbb{I}_j^{\text{vc}} \sigma^{\text{vc}} \left(1 - \xi_j^{\text{exit}}\right)\right\} \hat{q}_j dj + \int_{j \in J^{\text{entry}}} \hat{q}_j dj. \tag{12}
 \end{aligned}$$

There are several factors that contribute to the balanced growth rate. The first integral on the right-hand side of equation (12) captures the effect of surviving firms. Conditional on remaining in the business, intermediate firm  $j$  adds the step size  $\lambda$  if it generates an innovation, which happens at rate  $i_j$ . Moreover, if firm  $j$  issues public equity in the beginning of the next period, its quality

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<sup>23</sup>See Appendix A for the derivation.

increases by a factor  $\kappa - 1$ . The second integral captures the loss due to exiting firms. Notice that exit happens due to both the optimal decision of the firm and the attrition rate if the firm is matched with a VC. The last component of equation (12) denotes the contribution of entry.<sup>24</sup>

Finally, the following condition holds for the representative household.

**Proposition 1 (Euler Equation)** *In BGP, the household maximization implies the equilibrium interest rate  $r = (1 + g)^{\epsilon} / \beta - 1$ .*

### 3 Estimation

In order to measure the specific effects of different channels through which VC financing affects firm-level innovation and aggregate growth, I estimate the parameters of the model via the simulated method of moments (SMM). In this section, I first describe the identification and computation procedures. Then, I present the estimation results and discuss the goodness of fit. As a brief overview, the model successfully captures the duration of firm-VC matches and the firm age at the time of initial public offering as well as the aggregate patterns of R&D and growth. The model also replicates the difference in firm growth patterns between VC-backed and non-VC-backed firms observed in the data. I start by describing the parameters that are determined outside the model.

#### 3.1 Pre-determined Parameters

Because the model at hand is a fairly rich one with a large number of parameters, assigning some of them a priori mitigates the burden of estimation. There are 10 parameters that are chosen externally. The time period in the model corresponds to 1 year in the data. On the household side, the period utility function is assumed to have logarithmic form such that the curvature of the CRRA utility function,  $\epsilon$ , equals 1, the midpoint between various estimates surveyed in [Mehra and Prescott \(1985\)](#). The discount rate,  $\beta$ , is picked to imply a reasonable long run interest rate level, given the targeted rate of growth of 2%. Setting  $\beta = 0.98$  implies approximately a 4% real interest rate. On the final good production, the share of intermediate goods,  $\alpha$ , is set to 0.825. [Akcigit et al. \(2013\)](#) assign a value of 0.85 to physical factors in their final good production function. Without loss of generality, the marginal cost of producing intermediate goods is normalized to  $(1 - \alpha)$  for private firms that do not have VC support.<sup>25</sup>

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<sup>24</sup>Note that the entrant firms do not contribute through IPO because the support of the distribution from which they draw the initial product quality does not extend over values that lead to IPO.

<sup>25</sup>This normalization simplifies the derivation of the profit function. The corresponding value for VC-backed and public firms,  $\eta$ , is determined in the estimation.

The function  $h(\cdot)$  that defines the cost of doing R&D is assumed to have the form  $\gamma_0 x^{\gamma_1}$ . The curvature parameter,  $\gamma_1$ , is set to 2 so that the function has quadratic shape. This in turn implies that the R&D elasticity in the innovation production function is 0.5, a value in line with the empirical literature.<sup>26</sup> The lowest product development efficiency,  $\theta^L$ , is normalized to 1.

The parameter that governs the exogenous separation of matches between firms and VC funds,  $\sigma^{vc}$ , is set as follows. In [NVCA \(2013\)](#), the National Venture Capital Association (NVCA) reports that, among VC-backed firms that received their first round of funding between 1991-2000, about 16% made it to the IPO stage. Another 18% are reported to fail. The rest of the matches end in ways that I do not include in my model.<sup>27</sup> The exogenous separation parameter,  $\sigma^{vc}$ , captures the yearly attrition rate due to these external reasons.<sup>28</sup> For the average share sold at IPO,  $\Delta$ , [Ritter \(1998\)](#) reports a range of 20%-40%. The telegraph matching function introduces a single scale parameter that is normalized to 1. Lastly, the bargaining power of the VC,  $\phi$ , is assumed to be 0.5. Table (1) summarizes the predetermined values.

Table 1: Parameters Fixed A Priori

Value	Description	Source
$\beta = 0.98$	Discount Rate	Real Interest Rate
$\varepsilon = 1$	CRRA curvature	<a href="#">Mehra and Prescott (1985)</a>
$\alpha = 0.825$	Share of physical factor	Profitability
$\eta^H = 1 - \alpha$	Cost of capital, high	normalized
$\gamma_1 = 2$	R&D cost elasticity	<a href="#">Blundell et al. (2002)</a>
$\theta^L = 1$	Product development efficiency, low	normalized
$\rho = 1$	Scale of telegraph matching	imposed
$\phi = 0.5$	Bargaining power	imposed
$\sigma^{vc} = 8.7\%$	Attrition rate of VC-firm matches	Unsuccessful separations
$\Delta = .28$	Share sold at IPO	<a href="#">Ritter (1998)</a>

### 3.2 Identification of the Estimated Parameters

There are 10 parameters to be estimated. Perhaps the most crucial parameters are  $\{\theta^L, \theta^H\}$  because they determine the magnitude of the impact on firm growth of the operational knowledge provided by VCs. Having normalized  $\theta^L$  to 1, I make use of [Puri and Zarutskie \(2012\)](#) in estimating  $\theta^H$ . [Puri](#)

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<sup>26</sup>Measuring innovations by patents, the empirical literature on patents and R&D provides estimates for this elasticity. [Griliches \(1990\)](#) gives a range from 0.3 to 0.6 while [Blundell et al. \(2002\)](#) find 0.5.

<sup>27</sup>Among these remaining matches, half of them resulted in acquisition of the private firm by another one. The other half is counted as “still private or not known”, and most of them are believed to have failed. Because the success of a VC firm is generally measured by its IPO performance, I focus on IPOs.

<sup>28</sup>The total attrition rate is assumed to be the cumulative hazard rate over 7.5 years. This length of time represents the median tenure of VC investments, which is estimated to vary from 7 to 10 years in the data.

and Zarutskie (2012) make a fundamental contribution to the empirical literature that investigates the effect of venture capital on firm dynamics by employing survey data on firms. In particular, they combine the VentureXpert and Longitudinal Business Database of US Census Bureau so that they are able to determine the firms that received VC financing. Controlling for the number of employees, age, geographical location, and the industry at four-digit SIC level, they create a matched sample of non-VC-backed firms and firms that are at the first round of getting VC funding.<sup>29</sup> The authors observe the firms in these two categories until they exit in some form (exit the data, become public, etc.) for a maximum of 10 years. Then, using these samples, they regress the logarithm of firm sales on a number of covariates and, in particular, provide the OLS estimate on the interaction term between a dummy for VC use and the time elapsed after matching.<sup>30</sup> This estimate determines the differential impact of VC financing on firm growth. To determine  $\theta^H$ , I create analogous samples from the stationary distribution of my model. I simulate firms in these samples for 10 years, and conduct the same regression analysis. The estimation procedure tries to match the model counterpart of the OLS estimate with the one provided Puri and Zarutskie (2012).

The size of VC firms' financial impact is governed by the difference between  $\eta$  and  $\eta^H$ , the marginal costs of production for private firms with and without VC backing, respectively. To discipline this difference, I assume that the decline in the cost of capital due to VC investment reflects all the pecuniary support of VC companies. Then, including the ratio of VC investment to GDP as one of the data moments determines the size of this financial support in my model.<sup>31</sup>

In order to complete the estimation of the VC market, the entry cost for VC firms needs to be determined. The entry cost of the venture capitalist,  $\chi^{vc}$ , creates a threshold for the intermediate good qualities above which VC firms would not agree to form a match with a firm, because they could not generate a great deal of improvement on already high quality levels due to decreasing returns to innovations. Moreover, this entry cost determines the ex-ante value of a venture capitalist before entering the market. Therefore, once the other parameters that describe the matching function and Nash bargaining are fixed,  $\chi^{vc}$  is closely correlated with the probability of firms obtaining VC financing. Hence, to discipline  $\chi^{vc}$ , I include as a target the NVCA (2013) estimate that, roughly, only one out of a hundred applications succeeds in securing VC financing. One caveat: in my model, every meeting in the matching market results in a match. This happens both because there is no search cost for the firms, and because VC firms are identical. Any firm that knows that a match would create a positive surplus goes into the market, and the ones with the expectation

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<sup>29</sup>It should be emphasized that Puri and Zarutskie (2012) do not control for the amount of VC investment received, and do not uncover particular mechanisms through which VC affects firm dynamics. One contribution of my paper is to establish this.

<sup>30</sup>The OLS regression results are presented in Appendix B.

<sup>31</sup>Venture capital investments do not only include funding of early- and growth-stage companies, but also buyouts, later-stage investments, etc. not relevant to the point of this paper. Therefore, when calculating the ratio of VC investment to GDP, I take into account only the early- and expansionary-stage investments by VC.

of a negative surplus stay out. To map the NVCA statistic to my model, I interpret the 1% success on applications as the chance of meeting a VC company that would accept the firm. When solving the model, I fix the probability of matching with a VC at this level, and solve for the level of entry cost that supports the equilibrium by looping over  $\chi^{vc}$ .

To complete the cost structure of an IPO, the fixed cost of IPO,  $\chi^{IPO}$ , needs to be determined. This parameter maps to direct costs of IPO observed in the data, such as registration fee and underwriting costs. The statistics provided by Ritter (1998) indicate that, on average, these costs amount to 11% of the total proceeds raised by IPO. Using this figure, I can directly estimate  $\chi^{IPO}$ .

The benefits of an IPO are determined by two parameters:  $\kappa$ , the abrupt increase in quality level, and  $\theta^M$ , the permanent product development efficiency that VC-backed firms retain after becoming public. Determining the gains from IPO, these parameters are crucial for the decision of the optimal time to go public. To pin down  $\kappa$  and  $\theta^M$ , I therefore use the median age across all private firms at the time of IPO, together with the median length of firm-VC matches that lead to an IPO. Because product development efficiency is assumed to remain fixed for non-VC-funded private firms after going public,  $\kappa$  is the only parameter that determines the gains from going public for these type of firms. Then, the median time to IPO for VC-backed companies is helpful primarily in identifying  $\theta^M$ . Both  $\kappa$  and  $\theta^M$  are negatively related to these age moments.

The rest of the parameters are  $\lambda$ ,  $\gamma_0$ ,  $\chi^o$ , and  $\chi^e$ . The first one determines the quality gain due to process innovations and is mostly tied to the average growth, for which the target value is the average US post-war annual growth rate of 2%. The scale parameter of R&D cost function  $\gamma_0$  is used to match the R&D share of GDP. The outside option for intermediate firms,  $\chi^o$ , is estimated by targeting a 5.5% equilibrium exit rate. I take this value from Lee and Mukoyama (2012), who calculate estimates using US plant-level data from 1972-1997. I set the entry cost,  $\chi^e$ , that the potential entrepreneurs face, such that the equilibrium measure of intermediate good firms is equal to unity. As a result, the set of 11 parameters to be estimated within the feasible set  $\Omega$  is

$$\omega \equiv \left[ \eta, \gamma_0, \chi_{vc}, \rho, \chi_{IPO}, \kappa, \theta^M, \theta^H, \lambda, \chi_e, \chi_o \right]^T \in \Omega.$$

### 3.3 Algorithm

The computation of general equilibrium given a parameter set  $\omega^{given} \in \Omega$  consists of two nested fixed point problems. The outer loop searches for convergence on the growth rate. Given the growth rate, the inner loop computes the value functions. Computation of the value function for non-VC-backed firms requires another nested fixed point solution in the sense that the equilibrium matching rate and the value functions needs to be solved jointly. At this point, I modify the

problem so that I fix the matching rate at the targeted moment, and solve for the corresponding VC entry cost instead. This step requires calculation of the endogenous (normalized) quality distribution across firms. The reason is that, given the fixed matching rate, I use the value of the VC firm to update the guess for  $\chi^{vc}$ , and the value function of the VC firm depends on the endogenous distribution of firms searching for VC. To yield a smooth distribution, I discretize the possible values of the normalized quality levels into 1200 points for each firm type. Once I obtain the general equilibrium, I simulate samples from the stationary distribution to calculate the moments regarding the age of IPO for private firms, median duration to IPO in firm-VC matches, and the regression statistic that determines  $\chi^{vc}$ . Given a set of parameters  $\omega^{guess}$  the solution routine continues as follows:

1. Guess a growth rate  $\mathbf{g}^{guess}$ .
2. Solve for the value functions of
  - 2.i) Public firms
  - 2.ii) VC-backed private firms
3. Solve for the value function of non-VC-backed private firms.
  - 3.i) Guess a candidate entry cost,  $\chi^{guess}$ , for VC firms.
  - 3.ii) Compute the value function of non-VC-backed private firms.
  - 3.iii) With all value functions at hand, compute the stationary distribution.
  - 3.iv) Compute the implied  $\chi^{new}$  using the problem of the VC firm. Update until  $\|\chi^{guess} - \chi^{new}\| < \epsilon$ .
4. Compute the implied  $\mathbf{g}^{new}$ . Update until  $\|\mathbf{g}^{guess} - \mathbf{g}^{new}\| < \epsilon$ .

### 3.4 Estimation Results

#### Parameter Estimates

Table 2 reports the values for the parameter estimates obtained via the computation algorithm introduced above.

A number of parameter estimates in Table 2 merit special attention. The first variable in the table,  $\eta$ , determines the magnitude of the financial help of VC firms. The estimated value implies that VC firms decrease the marginal cost of intermediate good production by 6%. The economic meaning of this estimate is better reflected in the resulting difference in operation profit

Table 2: Estimated Parameters

Value	Description	Identification
$\eta = 0.94 \cdot \eta^H$	Cost of capital ratio	VC investment/GDP
$\gamma_0 = 50$	R&D cost scale	R&D investment/GDP
$\chi^{vc} = 0.435$	VC entry cost	Success in due diligence
$\chi^{ipo} = 0.20$	IPO fixed cost	Direct cost of IPO
$\kappa = 1.60$	Quality jump, IPO	Median duration with VC
$\theta^M = 4$	Efficiency, after VC	Median age at IPO
$\theta^H = 5$	Efficiency, with VC	Puri&Zarutskie (2012)
$\lambda = 0.275$	Innovation size	Growth rate
$\chi^e = 2.43 \cdot 10^{-4}$	Entry cost	Fixing measure to unity
$\chi^o = 6.06$	Outside option	Exit rate

levels. A back-of-the-envelope calculation shows that the estimated reduction in marginal costs translates into 30% higher operational profits for a VC-backed company compared to a non-VC-backed counterpart with the *same* product quality.

Two other important parameters are  $\theta^H$  and  $\theta^M$ , which, respectively, measure the direct and permanent (post-IPO) efficiency gains in product development due to VC firms' operational knowledge. The former implies that a VC-backed firm is five times more efficient than its non-VC-backed counterpart in improving a certain quality level with innovation intensity. Moreover, the estimate for  $\theta^M$  implies that the VC-backed firm retains 80% of this efficiency gain after going public. As the counterfactual experiments reveal below, this limited loss of efficiency even after separation from the VC firm has important implications for the effect of VC financing on long-run economic growth.

### Goodness of Fit

Table 3 summarizes the moment targets and their counterparts in the model. First of all, the model is successful in matching the aggregate growth rate and the ratio of R&D investment to GDP. Because innovation and aggregate growth are integral parts of the analysis, it is critical that the model reflects these aspects of the data well.

Looking at the data moments that largely define the VC market, the first result is that the model accounts for a fair amount of VC investment in the data. Notice that the implied parameter estimate results in a sizable improvement in operational profits of VC-backed companies, as explained above. Thus, it is fair to conclude that the estimation allows the monetary aspect of VC financing to have a significant impact on firm dynamics. The other channel, operational knowledge, has both a direct and a permanent effect on the firms that receive VC support. The direct

effect is disciplined by the regression statistic obtained from the analysis of Puri and Zarutskie (2012), and the model proves to be successful in matching this crucial target.<sup>32</sup> Moreover, the median duration of VC-backed firms until IPO in the model mirrors the data target very closely. Matching this target is important because it disciplines the permanent effect of VC's operational knowledge as well as the IPO cost for VC-backed firms in this regard.

Table 3: Model Fit

Target	Data	Model
VC investment/GDP	0.17%	0.12%
IPO direct costs	11%	6.26%
Match probability	1%	1%
R&D investment/GDP	2.8%	2.89%
Median duration with VC	5.5yrs	6yrs
Median age at IPO	12yrs	11yrs
Regression statistic	0.212	0.242
Growth rate	2.0%	1.95%
Exit rate	5.5%	2.75%

<sup>1</sup> The regression statistic provided by Puri and Zarutskie (2012) is highly significant with a t-statistic 11.23.

### Non-targeted Moments

Table 4 reports statistics observed in the data and not targeted in the estimation of the model, together with their data counterparts.

Table 4: Non-targeted Moments

	Data	Model
IPO probability of VC-backed firms	16%	20%
VC-backed IPOs	50%	51%
Output share of private firms	46%	48%
Relative firm growth	0.75	0.22

First of all, the model captures the IPO patterns in the data accurately. The model simulations based on samples of 50,000 firms imply that about 16% of VC-firm matches end up with an IPO. This number is the ballpark of the value found in the 2013 report of the National Venture Capital Association (NVCA, 2013). For the private firms without VC support, the corresponding value is

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<sup>32</sup>I discuss the implications for firm growth below.

1.7% in the model. This is well below the value for VC-backed companies, a pattern also observed in the data. Similarly, the fraction of IPOs involving VC-backed firms is also closely reflected in the model. The recent IPO report by [WilmerHale \(2014\)](#), a widely recognized law company in the US, documents that in 2013, VC-backed IPOs constituted half of all IPOs, whereas the data statistics in [Ritter \(2014\)](#) indicate that an average of 38% of IPOs were VC-backed between 2006-2013.

A closer look at the firm type composition of the model economy shows that 48% of output is produced by privately held firms. The most recent figures from the U.S. Small Business Administration data similarly show that a little less than 50% of the US GDP is produced by firms with fewer than 500 employees of which almost all are private firms ([Kobe, 2012](#)).<sup>33,34</sup> Regarding the growth rates of private firms, the estimated model predicts that the yearly average growth rate of VC-backed firms is 22% higher than the rate of the non-VC-backed sample.<sup>35</sup> The corresponding figure in the data is obtained from [Puri and Zarutskie \(2012\)](#). As explained in detail, [Puri and Zarutskie \(2012\)](#) explore growth rates of different samples of private firms with and without VC backing that are matched based on observable characteristics. They document that, over the first 10 years after the time of matching, the average growth rate of the VC-backed sample is 75% higher. Although at a smaller magnitude, the model captures this pattern qualitatively. This smaller magnitude indicates that the model provides a lower bound for the VC impact observed in the data.<sup>36</sup>

Figure 2 shows the impact of VC on firm distribution over a 10-year period. Following [Puri and Zarutskie \(2012\)](#) I create a model sample of private firms from the stationary distribution that defines new matches with VC firms. The thin solid line shows this initial distribution. I then simulate two versions of this sample across 10 years. In one version, firms are assumed to receive VC financing whereas in the other, firms continue without VCs and are observed until they obtain VC, issue public equity or exit the market. Starting the simulation with the identical group of firms replicates the matching exercise of firms in the data based on their sales, as done by [Puri and Zarutskie \(2012\)](#). The resulting difference between VC-backed and non-VC-backed samples after 10 years is illustrated by the dashed and thick solid lines, respectively. Among the VC-backed firms that remain after 10 years, there is a population of firms that survive with lower sales and

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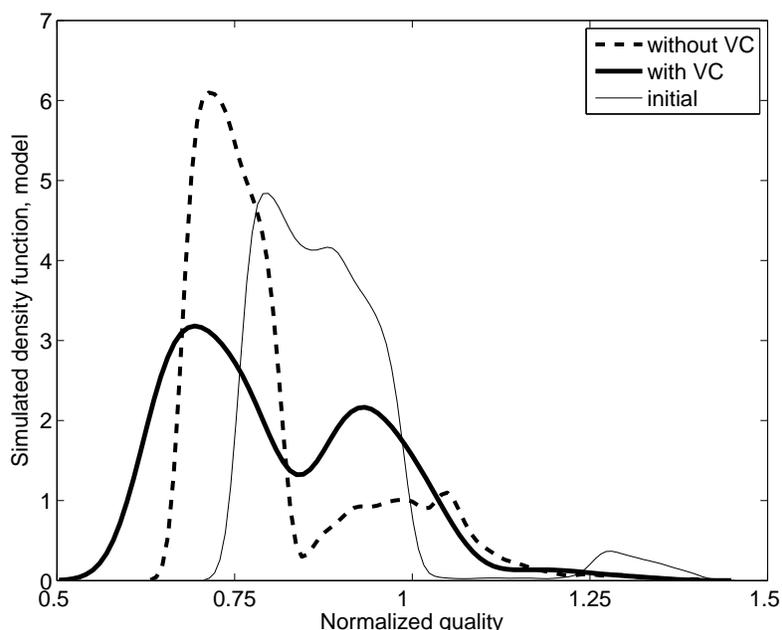
<sup>33</sup>It is true that a small fraction of firms in the economy are public, and most of the large firms with more than 500 employees are private. However, given that my focus is on the dynamics of young and innovative firms as opposed to very large private firms, matching the output share of firms with less than 500 employees is a reasonable comparison.

<sup>34</sup>[Asker et al. \(2014\)](#) report that all private firms account for 59% of sales.

<sup>35</sup>In this exercise, firm growth is defined as sales growth, in line with [Puri and Zarutskie \(2012\)](#).

<sup>36</sup>This smaller magnitude can be partially attributed to the exogenous attrition process that hits every VC match with the same probability, i.e., it destroys successful matches at the same rate as it does relatively unsuccessful ones. However, in reality, an important share of the exits that the attrition rate accounts for in the model are unsuccessful firms. Therefore, the figure of 22% generated by the model can be considered as an attenuated value for the growth rate differential between VC-backed and non-VC-backed samples.

Figure 2: Evolution of Matched Samples in the Model



profits. Notice that, in the model, the value of the outside option,  $\chi^o \bar{q}_t$ , is the same for any firm type. This shows that financial support from VCs through lower intermediate production costs helps some firms with a lower productive capacity remain in the economy. However, as the last row of Table 4 shows, the yearly average growth rate of the VC-backed sample is 22% higher in the model. This impact is reflected by the fatter right tail of the resulting distribution of the VC-backed sample. VC firms' operational knowledge enables a larger subgroup of firms to achieve higher levels of production compared to the non-VC-backed sample. This outcome is in line with the reality that many portfolio companies of VC funds are relatively unsuccessful, while a few perform exceptionally.

Regarding VC impact on firm growth, one caveat is worth mentioning. Despite the fact that Puri and Zarutskie (2012) controlled for observable characteristics when creating matched samples, this procedure did not account for a possible selection of firms by VC companies according to unobservable features. Suppose that there was sorting of firms that are superior on some unobservable quality towards VC investment. If the matching procedure does not account for this type of sorting, and if that affects firm growth positively, then this would inflate the apparent effect of VC investment through operational knowledge in my model, since the contribution of this unobserved quality would be inaccurately assigned to that channel. It is fundamentally important to notice that this *would not* bias my estimation because it proceeds on the method of indirect inference, replicating the same empirical experiment as in Puri and Zarutskie (2012). Nonetheless, when interpreting the impact of operational knowledge in both the model and in the data, the potential effect of selection on unobservables can be included, using the findings of Sørensen

(2007). Using data on IPO rates of VC-backed companies, [Sørensen \(2007\)](#) shows that the portfolio companies of more experienced VCs are more likely to go public. Then, he structurally estimates a two-sided matching model to find that sorting, defined as the fact that more experienced VCs invest in better firms, accounts for 50% to 60% of the higher IPO rate in companies backed by more experienced VCs. In other words, the direct influence of VC on the firm is 40-50%. This estimate, however, reflects the differential effect of VCs' expertise *only* across VC-backed firms, and does not account for its significance in comparison to firms that completely lack VC backing. This means that it attenuates the relevance of VCs' direct influence on firms. Nevertheless, if a conservative path were followed based on [Sørensen \(2007\)](#), the estimate of 22% that my model implies for VCs' impact on firm growth would still translate into 10%.

## 4 Quantitative Exploration

Having estimated the parameters of the model and analyzed the model fit, I use this framework for two purposes. First, I measure the significance of VCs' operational knowledge relative to the financing channel, in terms of the aggregate growth of the economy. To do so, I run counterfactual experiments in which I marginally increase the parameters that govern financing and the operational knowledge channels. I then compare the resulting changes in the growth rates of the economy. Next, I replicate a recent policy measure that the European Union has adopted to make the investment environment more hospitable for venture capital firms. In the model, I capture the essence of the policy by decreasing the entry cost of VC firms, and explore the impact on long-run economic growth.

### 4.1 Counterfactual Analysis: Strength of Operational Knowledge

To measure the relative impact of the operational knowledge channel in aggregate growth terms, I first consider a hypothetical economy in which I increase the parameters  $\{\theta^M, \theta^H\}$  by 5% without changing  $\theta^L$ .<sup>37</sup> I then run a similar experiment where I increase the size of the marginal cost reduction due to VC help by the same amount, keeping the other parameters at the estimated levels. These experiments allow me to compare the elasticity of the growth rate to the distinct channels through which VC firms affect firm dynamics.

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<sup>37</sup>One alternative approach to measure the impact of operational knowledge is to consider its absence by removing the increase in product development efficiency due to VC support. The reason I do not use this approach is that in this case no private firm searches for VC. Nevertheless, this endogenous shutting down of the market happens even when there are small efficiency gains from VC help. In that case, removing the efficiency gain completely does not measure the exact impact of operational knowledge on aggregate growth.

Table 5: Counterfactual Experiments

	Benchmark	$\{\theta^M, \theta^H\}$ 5% higher	$(\eta^H - \eta)$ 5% higher	No VC	IPO fixed cost 15% higher	VC entry cost 3.5% lower
Growth	1.95%	2.01%	2.06%	1.39%	1.93%	2.02%
Measure of firms	1	1	1.16	0.99	1	1.01

The first three columns of Table 5 summarize the response of the economy to marginal changes in different aspects of VC support in comparison to the estimated economy. The table also reports the equilibrium measure of intermediate firms because the changes also affect the endogenously determined size of the economy. Comparing the growth rates in the second and third columns shows that the marginal increase in the strength of the operational channel leads to a 0.06% gain in the growth rate, whereas this number is around 11% when the change in the financial channel is considered. Therefore, the main message of this comparison is that, in terms of long-run growth, the operational channel is about half as effective as the financial channel. In other words, the influence of the operational knowledge channel on growth through its impact on firms accounts for about 1/3 of VCs' total contribution to aggregate growth through its impact on firms. The increase in financial impact also expands the equilibrium measure of products by 16%, in addition to its effect on long-run growth.

A deeper look into the hypothetical model economies reveals that, in the case of stronger financial support, most of the impact is generated through the changing composition of private firms. In the hypothetical world with increased financial impact, the ratio of output produced by VC-backed firms to the amount produced by all private firms is about 50%, whereas this number is about 4% in the estimated actual economy.<sup>38</sup> One reason for this result is that, due to the higher aggregate growth rate, the fixed cost of IPO, which is proportional to aggregate productivity, increases faster. This leads to longer durations of VC matches before VC-backed firms go public.<sup>39</sup> In the case of stronger operational knowledge influence, however, these fluctuations are much more limited. Instead, the impact on long-run growth of the economy stems from the increased efficiency of development, both for the VC-backed firms and for the public firms that received VC support.

Lastly, the fourth column in Table 5 implies that, in a hypothetical world without a market for venture capital, the growth rate would go down to 1.40%. Here, I assume that all firms operate with low efficiency, and there is no means to affect it. This lower efficiency, in turn, leads to a drastic

<sup>38</sup>Notice that the 5% additional reduction in marginal cost translates into more than 25% additional profits per unit of product quality. The huge responses in the hypothetical economy stem from this fact. Thus, it is plausible to think that these counterfactual comparisons provide lower bounds for the relative importance of the operational knowledge channel.

<sup>39</sup>The resulting median age is around 9 years, whereas in the benchmark economy this was 6 years.

fall in the growth rate. Regarding this experiment, one caveat is that all firms operate at the higher marginal cost of production because the only financial intermediary in the model is removed. This has an indirect growth impact because higher profits due to VCs' financial support create an indirect incentive for innovation due to a larger return per unit of product quality. The drastic fall in the aggregate growth rate would potentially be smaller if there were an alternative intermediary with a similar financial impact. Therefore, this result should be interpreted cautiously.

## 4.2 Counterfactual Analysis: Higher Cost of IPO

A widely held belief is that there are strong complementarities between VC finance and an active public equity market (Black and Gilson, 1998; Michelacci and Suarez, 2004).<sup>40</sup> The intuition is the following: on the one hand, VC firms accelerate new ventures towards issuing IPO, through the aforementioned influences. On the other hand, liquid stock markets provide an attractive and affordable IPO option for private firms, and a profitable way for VC firms to separate from portfolio companies.

To analyze the linkages between public equity issuance and VC financing in my model, I now consider an economy in which the fixed cost of IPO is 15% higher. I obtain this value from Kim et al. (2003). In their study of equity and debt issues from 1970s to 2000s, Kim et al. (2003) document that average underwriting spreads for IPOs in the US were 8.5% and 7.4% over the periods 1976-1985 and 1996-2005, respectively.<sup>41</sup> Having used data from the latter period in my estimation, I now analyze the counterfactual setting where I set the fixed cost of IPO to its earlier value. As shown in the fifth column of Table 5, the growth rate falls by about 1.5 basis points.

Figure 3 shows the resulting changes in the IPO decisions. The increase in the fixed cost affects the IPO threshold on the firm size negligibly for VC-backed firms whereas the threshold for non-VC-backed firms rises discernibly. The difference stems from the higher profit levels that VC-backed firms generate with the same quality level due to VCs' financing support. The higher threshold in turn implies a 3% fall in the non-VC-backed equity issuances. Therefore, it expands the group of private firms available to match with a VC. Together with the VC firms' willingness to search longer due to the higher equilibrium discount factor, this change means a 10% lower probability that a private firm will match with a VC.<sup>42</sup> In turn, the share of VC-backed firms in the economy decreases by 11%, which also lowers the share of public firms that had received VC support when they were private. In combination, all these responses result in a 1.5 basis point loss

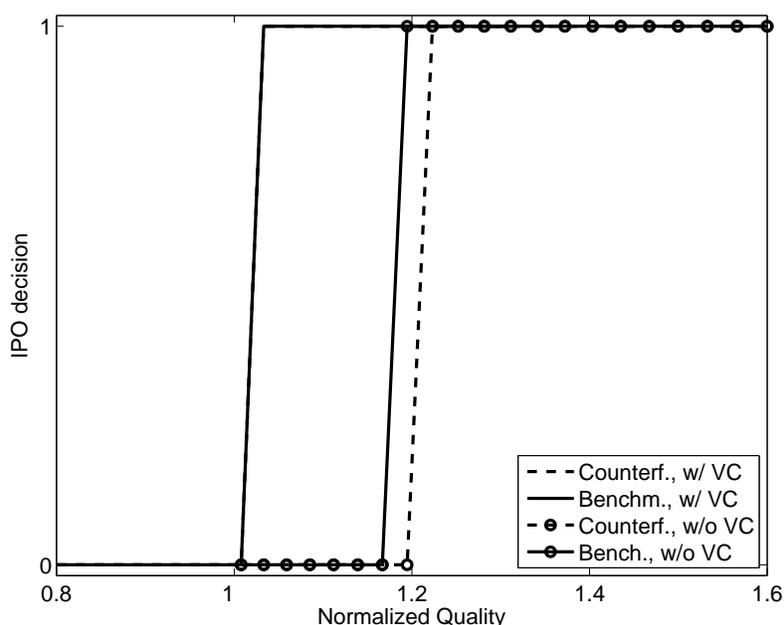
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<sup>40</sup>Bottazzi and Rin (2002) report supporting evidence on the positive impact of Euro.nm, the European counterpart of Nasdaq, on European VC activity in 1990s. Euro.nm, opened in 1997, is the alliance of new European stock markets that focuses on growth companies.

<sup>41</sup>The underwriting costs are one of the main determinants of the fixed costs associated with IPOs.

<sup>42</sup>A lower growth rate implies a lower interest rate; therefore, VC firms discount the future value less.

Figure 3: IPO decisions



in the long-run growth rate of the economy.

### 4.3 Policy Analysis: VC Entry Cost

In 2013, the European Union adopted a new regulation on venture capital funds to enhance funding to small and medium businesses through venture capital financing.<sup>43</sup> As a main obstacle to adequate VC funding, the European Commission recognized the lack of a harmonized VC market across the Union. According to the Commission, the fragmented structure of the VC market across national borders increases VC firms' costs due to changing national regulatory environments, especially regarding the raising of capital. To help VC firms expand their operations by easing fundraising, the EU passed a new regulation that introduces a designation called the "European Venture Capital Fund". VC managers whose funds meet certain requirements, such as high concentration on investment in young and innovative companies, can raise capital under this rubric and be subject to a single rulebook across all EU countries.<sup>44</sup>

To analyze the potential effects of this regulation, I interpret the lower fundraising cost for VC firms through the lens of my model as lower entry costs. In the experiment, I assume that 3.5% of the entry cost is subsidized through a lump-sum tax on the representative household.<sup>45</sup> As the

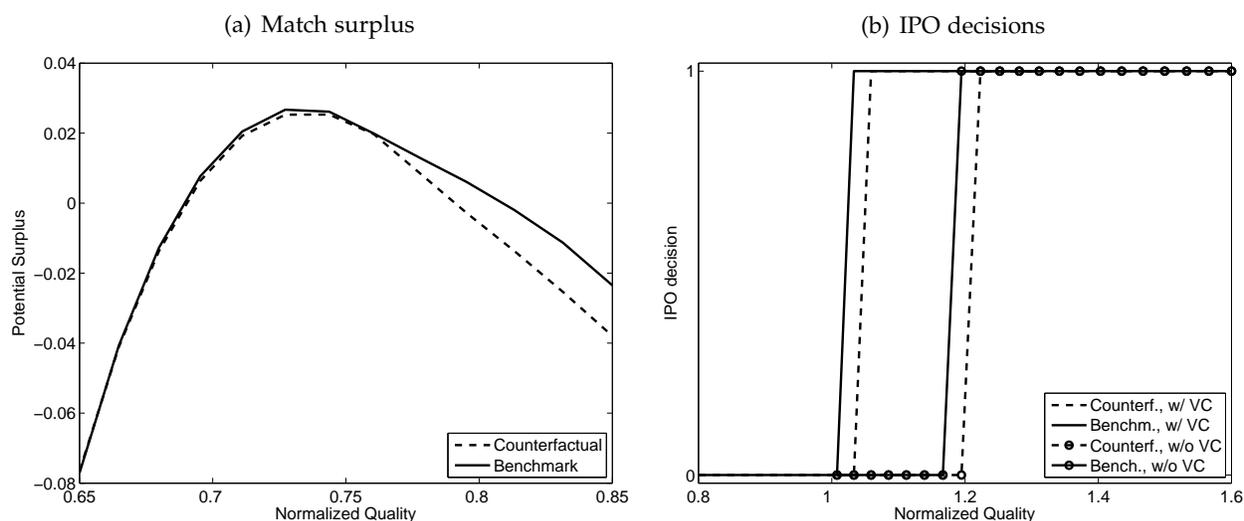
<sup>43</sup>The legislative act by the [European Commission \(2013\)](#) explicitly recognizes that "venture capital funds provide undertakings with valuable expertise and knowledge, business contacts, brand equity and strategic advice".

<sup>44</sup>The law requires qualifying funds to channel at least 70% of their capital to small and medium enterprises (SME).

<sup>45</sup>This value has been picked such that, in the new equilibrium, a reasonably low share of output is used to finance the subsidy.

last column of Table 5 shows, this subsidy increases the long-run growth rate by around 7 basis points. A back-of-the-envelope calculation illustrates that, in equilibrium, the cost of this policy is 0.09% of output. Correspondingly, the subsidized amount is about 8% of the total investment made by VCs into portfolio companies in the benchmark economy.

Figure 4: Response to lower VC entry cost



The details of how the economy responds to the subsidy show similarities to the counterfactual experiment in which the financial support of VC was expanded. Through the general equilibrium effects, there is a reallocation of private firms towards being VC-backed companies, and these stay with the VC firm for a greater period of time. This longer match duration amplifies VCs' effect on the aggregate growth rate through the influence of superior operational knowledge. This happens due to two factors. First, as illustrated in Figure 4(b) for both VC-backed and non-VC-backed firms, there is an increase in the (normalized) quality thresholds above which the two types private firms issue IPO. This shift emerges, again, due to a combination of the proportionality of the fixed IPO cost to average productivity, and the higher resulting equilibrium aggregate growth rate. As a result, firms need to develop their product quality further to afford IPO issuance costs. The second factor for increased match durations is that VC firms match with companies of smaller quality levels, which have larger potential for growth. For a given IPO threshold, this implies that, on average VC-backed firms have to innovate more to reach the IPO stage. Figure 4(a) delineates this point. The curves show the present discounted value of the surplus of a potential match between a VC firm and private firms with quality  $\hat{q}_j$ . In this economy with subsidies, this curve shifts towards the left so that a positive surplus, and thus a profitable match, is possible with firms of smaller size. These combined changes result in a 7 basis point higher long-run growth

rate of the economy.<sup>46</sup>

## 5 Conclusion

Motivated by the disproportionate investment of venture capital finance in young and innovative businesses, I study in this paper the quantitative impact of VC financing on firm dynamics and economic growth. I propose a new dynamic general equilibrium model of innovation with heterogeneous firms by introducing an explicit venture capital (VC) market. The model allows me to conduct counterfactual experiments which I use to quantify the impact of VC financing and examine relevant policies. I pay particular attention to a unique feature of VC firms that is largely overlooked by current macroeconomic analysis: the operational knowledge that VC firms bundle with their cash investment. In the model, technologically heterogeneous firms engage in innovative activities to improve their product quality and increase their profits. The efficiency of this product development process can be enhanced through the operational knowledge of VC firms. The model also includes an endogenous search and matching setting where VC companies and firms meet. In this way, the model accounts for the selection aspect of the VC market in addition to the cash investment and the operational knowledge. This is crucial to capture general equilibrium effects.

I structurally estimate this model using US data on VC finance, public equity issuances, and research and development expenditures. I identify the operational knowledge channel through its distinct impact on firm growth. Out-of-sample tests demonstrate that the estimated model successfully captures the non-targeted data moments such as IPO frequency of VC-backed firms, and the differences in growth rates of VC-backed and non-VC-backed firms, among others.

I then use the estimated model to conduct counterfactual and policy analyses. First, I measure the impact of the operational knowledge channel in terms of aggregate economic growth. The analysis indicates that a sizeable fraction,  $1/3$ , of VCs' impact on economic growth is generated through operational knowledge channel. This result implies that, for financing innovation, VC has significant value beyond capital investment alone. Next, I evaluate a recent policy adopted by the European Union. The regulation aims to decrease fundraising costs for VC firms and expand VC investment across borders by harmonizing the relevant regulatory environment throughout the Union. I examine effects of a subsidy on VC entry cost that simulates the policy and find that this

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<sup>46</sup>In this economy, all private firms available for VC matches have a certain identical efficiency level. A heterogeneity in this margin could dampen the effect of the policy change because some of the new VC firms had to meet with firms that already have higher efficiency. A parallel impact could arise if there were heterogeneity across VC firms in their potential to affect product development efficiency (Hsu, 2004; Bottazzi et al., 2008). A similar concept of heterogeneous firm entry and its aggregate productivity implications is investigated by ?. Incorporating these margins of heterogeneity by deriving the relevant empirical distributions in the data and deploying them in the estimation procedure is an attractive area for future research.

change can generate a 7 basis point gain in the long-run growth rate of the economy.

This paper provides fruitful ground for several directions of future research. One immediate step could be to explore the implications of heterogeneity across VC firms and the sensitivity of VC impact on firm growth to this aspect. A broader research question would be how the VC market arises endogenously. The optimal provision of operational expertise by VC requires managers who possess both sufficient operational knowledge and financial wealth. Explaining the reasons why and how venture capitalists emerge could help us understand the vast differences in the size of VC markets across different regions, such as the US and continental Europe.

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# Appendices

## A Derivation of the Growth Rate

The average growth rate in the economy is equal to the growth rate of the average quality level  $\bar{q}_t$  whose value in  $(t + 1)$  becomes:

$$\int_{j \in J_{t+1}} q_{jt+1} dj = \bar{q}_{t+1} = \int_{j \in J_t} \left(1 - \xi_j^{exit}\right) \left\{ (q_{jt} + i_j \lambda \bar{q}_{jt}) + \xi_j^{ipo} (\kappa - 1) (q_{jt} + i_j \lambda \bar{q}_{jt}) \right\} dj + \int_{j \in J^{entry}} q_{jt} dj$$

The components of this expression take into account the changes due to innovation, IPO, exit, and entry, as explained in detail in Section 2.5. Dividing both sides of this expression by  $\bar{q}_t$  and dropping time subscripts in BGP, we obtain

$$\begin{aligned} 1 + \mathbf{g} &= \int_{j \in J^*} \left(1 - \xi_j^{exit}\right) \left\{ (\hat{q}_j + i_j \lambda) + \xi_j^{ipo} (\kappa - 1) (\hat{q}_j + i_j \lambda) \right\} dj + \int_{j \in J^{entry}} \hat{q}_j dj \\ &= \int_{j \in J^*} \left(1 - \xi_j^{exit}\right) \left\{ \hat{q}_j + \xi_j^{ipo} (\kappa - 1) \hat{q}_j \right\} dj + \\ &\quad \int_{j \in J^*} \left(1 - \xi_j^{exit}\right) \left\{ i_j \lambda + \xi_j^{ipo} (\kappa - 1) i_j \lambda \right\} dj + \int_{j \in J^{entry}} \hat{q}_j dj \\ &= \int_{j \in J^*} \hat{q}_j dj + \int_{j \in J^*} \left\{ \xi_j^{ipo} (\kappa - 1) \hat{q}_j - \xi_j^{exit} \left[ \hat{q}_j + \xi_j^{ipo} (\kappa - 1) \hat{q}_j \right] \right\} dj + \\ &\quad \int_{j \in J^*} \left(1 - \xi_j^{exit}\right) \left\{ i_j \lambda + \xi_j^{ipo} (\kappa - 1) i_j \lambda \right\} dj + \int_{j \in J^{entry}} \hat{q}_j dj. \end{aligned}$$

The second equality above collects the normalized quality levels into the first integral. The third equality separates  $\int \hat{q}_j dj$  which, by definition of normalized quality, equals 1. Hence, we arrive at

$$\begin{aligned} \mathbf{g} &= \int_{j \in J^*} \left(1 - \xi_j^{exit}\right) \delta_j^{ipo} (\kappa - 1) \hat{q}_j dj - \int_{j \in J^*} \delta_j^{exit} \hat{q}_j dj + \\ &\quad \int_{j \in J^*} \left(1 - \xi_j^{exit}\right) \left\{ i_j \lambda + \xi_j^{ipo} (\kappa - 1) i_j \lambda \right\} dj + \int_{j \in J^{entry}} \hat{q}_j dj \\ &= \int_{j \in J^*} \left(1 - \xi_j^{exit}\right) \left\{ i_j \lambda + \xi_j^{ipo} (\kappa - 1) (\hat{q}_j + i_j \lambda) \right\} dj - \int_{j \in J^*} \xi_j^{exit} \hat{q}_j dj + \int_{j \in J^{entry}} \hat{q}_j dj. \end{aligned}$$

## B Firm Growth in Data

Table 6: Regression Results, [Puri and Zarutskie \(2012\)](#)

	Log(Sales)
VC	0.502*** (11.03)
VC*TimefromMatch	0.211*** (11.23)
VC*TimefromMatch <sup>2</sup>	-0.012*** (-8.73)
TimefromMatch	0.121*** (10.11)
TimefromMatch <sup>2</sup>	-0.003*** (-4.12)
Industry FE	Yes
Year FE	Yes
N	17,885
R <sup>2</sup>	0.18

Table 6 summarizes the OLS regression results obtained [Puri and Zarutskie \(2012\)](#) with t-statistics given in parentheses. The logarithm of sales of matched VC-backed and non-VC-backed samples is regressed on a number of independent variables and controls. The variable of interest that captures the effect of VC firms over time is “VC\*TimefromMatch”. It is highly significant with a t-statistics of 11.23.

## C Firm Size Distributions

Figure 5: Firm Size Distributions by Firm Types

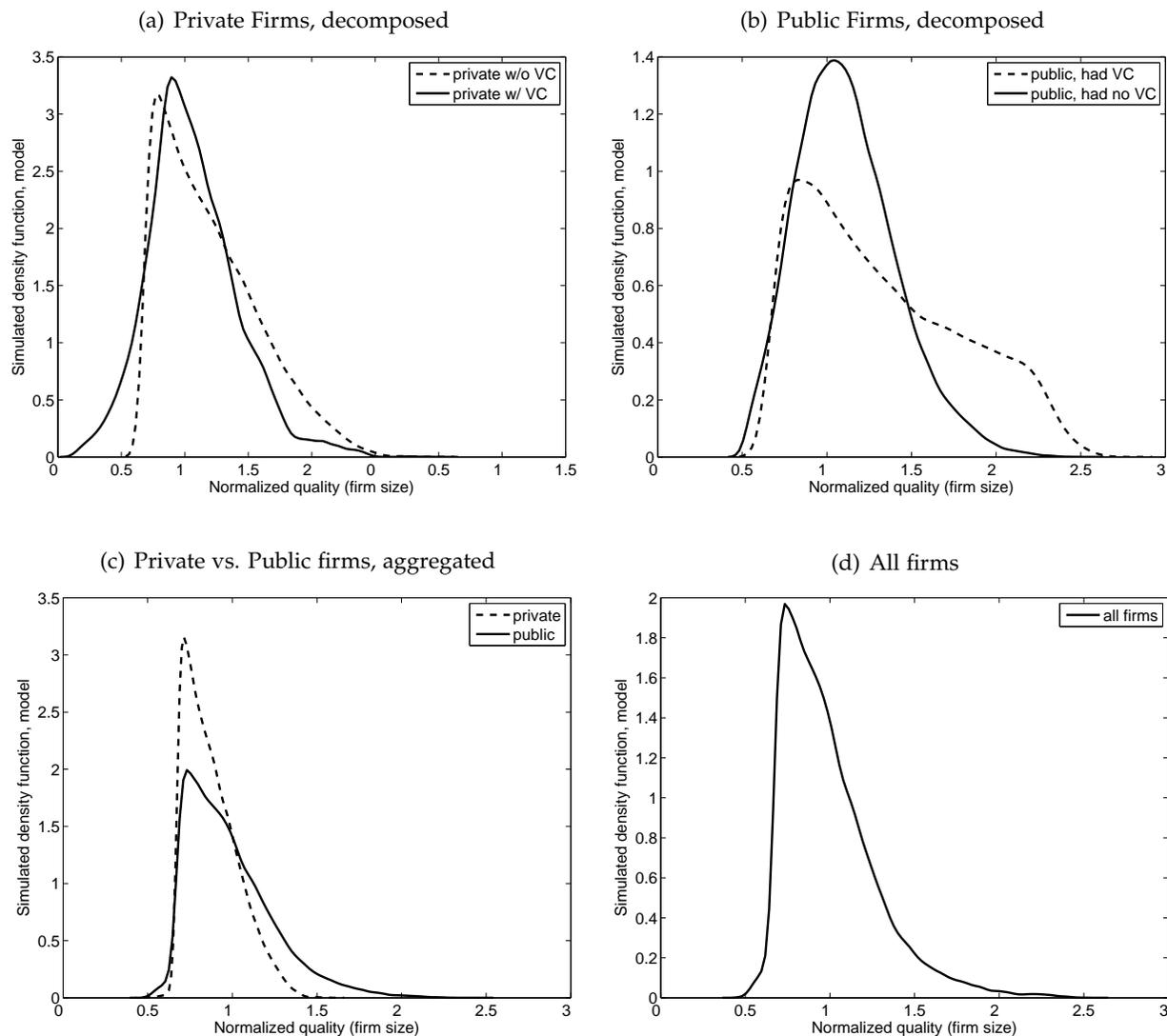


Figure 5(a) illustrates the stationary distributions of VC- and non-VC-backed private firms. The distribution of VC-backed firms has a larger mass of smaller companies compared to non-VC-backed counterparts. Although it may look counter-intuitive at first sight, this is a natural result of three factors.<sup>47</sup> First, VC firms select smaller companies to for their higher growth potential. Second, the increased profit level per unit of quality due to VC's financial support helps firms with smaller capacities survive in the business. And third, as demonstrated in Figure 4(b), VC firms go public at smaller sizes as they can afford its cost due to higher profits. This

<sup>47</sup>Notice that this is a static comparison. The comparison of growth rates in Section 3.4 has already explained the positive growth impact of VC financing on firms that they invest in.

implies that companies that are smaller than a relatively lower threshold remain in the VC-backed distribution.

The lower IPO threshold for VC-backed firms also implies that, every period, smaller firms enter the distribution of public firms via VC-backed IPOs. Therefore, as shown in Figure 5(b), the distribution that defines public firms that had VC backing has a thinner tail compared to the stationary distribution of public firms that never received VC support. Figure 5(c) compares the stationary distributions of all private and public firms. As expected, the latter has a fatter right tail because larger firms issue an IPO. Lastly, Figure 5(d) shows the stationary distribution of all firms in the economy.