Fewer but Better: Sudden Stops, Firm Entry, and Financial Selection*

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Abstract

We develop a tractable quantitative framework to study the productivity effects of financial crises. The model features endogenous productivity, heterogeneous firm dynamics, and aggregate risk. Selection of the most promising ideas gives rise to a tradeoff between mass (quantity) and composition (quality) in the entrant cohort. Chilean plant-level data from the sudden stop triggered by the Russian sovereign default in 1998 confirm the model's main mechanism, as firms born during the credit shortage are *fewer, but better* in terms of idiosyncratic productivity. The quantitative analysis shows that at the end of the crisis, total output is permanently 0.9% lower.

Keywords: Selection, Financial Crises, Endogenous Growth, Firm Dynamics

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1. Introduction

Most of the economic analysis of financial crises focuses on their short-run costs for the real economy. However, a body of empirical studies has documented persistent output losses following large economic downturns (Cerra and Saxena, 2008; Reinhart and Rogoff, 2014), pointing to permanent losses in total factor productivity (Meza and Quintin, 2007; Blyde et al., 2010). Because creative destruction—the process through which the entry of new firms and the expansion of incumbent firms drive other firms out of business—and the associated reallocation of resources are the main drivers of productivity growth (see Foster et al., 2001; Akcigit and Kerr, 2018), a potential determinant of this long-run cost of crises is fluctuations of entry and expansion decisions. In this paper, we propose a theory that links temporary shocks to permanent changes in aggregate productivity due to the endogenous response of the productivity enhancing decision of firms, and we use this framework to study the economic cost of financial crises.

A realistic theory of creative destruction and financial crises must recognize two forces. First, a financial crisis reduces the incentives for the creation of new firms and the expansion of existing ones.[1] However, a second force counteracts the first one: during crises, entrants are more positively selected, increasing the average productivity of the entering cohort. Therefore, it is unclear if fluctuations in creative destruction can generate quantitatively relevant permanent losses in aggregate productivity. We develop a quantitative model disciplined by micro data to show that creative destruction dynamics during financial crises can have sizable, long-lasting effects on productivity.

Our model bridges the gap between the small open economy business cycle (Mendoza, 1991; Neumeyer and Perri, 2005; Uribe and Yue, 2006) and the creative destruction literature of firm dynamics (Klette and Kortum, 2004; Lentz and Mortensen, 2008; Akcigit and Kerr, 2018), allowing for firm heterogeneity and selection.[2] In particular, we present a real business cycle small open economy model where productivity growth is endogenously determined by the dynamic decisions of heterogeneous firms. A financial intermediary has a portfolio of business ideas that can generate either high- or low-type firms. High-type firms give rise to drastic productivity improvement in the production technology of an intermediate product when they expand, while low-type firms give rise to only marginal improvements in the

[1] Firm-level decrease in productivity enhancement activities during financial crises has been documented by de Ridder (2017) and Duval et al. (2017).

[2] Throughout the text, firm dynamics refers to the continuous expansion and contraction of firms driven by their productivity-enhancing investments.

production technology. Every business idea is characterized by its idiosyncratic probability of giving rise to a high-type firm. Hence, business ideas are *ex-post* heterogeneous in terms of the productivity advantage that the new firm enjoys after entering the industry, and they are also *ex-ante* heterogeneous in their idiosyncratic probability of generating a high-type firm. Because only a handful of ideas have strong chances of generating high-type firms, good business ideas are scarce. The optimal allocation of funding follows a cut-off rule based on the idiosyncratic probability of becoming a high-type firm, which introduces a linkage between the size of the entrant cohort and the average efficiency gains generated by its members. The forward-looking decision of incumbents to acquire new products and the entry of new firms determine the endogenous rate of creative destruction and shape the evolution of the firm size distribution and the aggregate productivity in the economy.

The model economy features aggregate risk with stationary TFP and interest rate shocks. A financial crisis is modeled as a positive interest rate shock. A mass-composition tradeoff arises at the entrant cohort level. Episodes of high interest rates imply larger discounting of future profits increasing the selection incentives for the financial intermediary, giving rise to smaller cohorts with higher expected average productivity. Because of the future expansion decisions of the cohorts born during the crisis, composition dynamics persist long after the crisis has vanished. Moreover, the incentives for incumbents to expand are also affected during the crisis. On the one hand, lower entry rates decrease the treat of replacement by entrants. On the other hand, higher discounting and lower aggregate demand decreases the value of expansion for incumbents. On aggregate, incumbents also decrease the expansion efforts further contributing to the productivity slow down.

The empirical section studies a particular financial crisis, the Chilean sudden stop of 1998, to validate the tradeoff between mass and composition at the core of the model.[3] We focus on Chile for three reasons: (i) it is a small open economy; (ii) plant-level data for Chilean manufacturing firms are publicly available, and these data allow us to directly study entrant cohorts; and (iii) the sudden stop after the Russian sovereign default was essentially an exogenous shock to the Chilean economy (Calvo and Talvi, 2005). We document that the Chilean crisis lead to output and productivity losses that persist even eight years after the episode. Then, we show that firm entry in Chile decreased by 29% during the sudden stop. However, firms born in crisis are not just fewer, they are also better. In fact, the econometric analysis in Section 4 shows that, after controlling for individual characteristics, firms born

[3]A sudden stop in capital flows is a large and abrupt decrease in capital inflows, characterized by jumps in sovereign spreads and quick reversals of current accounts deficits.

during the sudden stop are, on average, 64% more productive (measured by revenue total factor productivity) during their life span than firms born during normal times.[4]

In the quantitative section of the paper, we calibrate the model to the pre-crisis Chilean economy. The balanced growth path (BGP) of the calibrated model matches non-targeted moments of the firm life cycle and dynamics. The stochastic solution of the model also matches non-targeted moments of the Chilean business cycle. We then use the Chilean sudden stop to assess the performance of the model during the crisis. To discipline the two exogenous shocks in the model, we target two series from the data. Precisely, we filter the stationary productivity component and the interest rate shocks using interest rate and manufacturing output data. The model captures successfully the dynamics of firm entry and the associated dynamic selection effects observed in the data during the crisis.

We use the calibrated model to quantify the productivity loss due to the sudden stop. The productivity and interest rate fluctuations in Chile between 1997–2002 generate a permanent productivity loss of 0.9%. Therefore, absent any further events, the Chilean economy would converge to a new path where output and consumption are permanently 60 basis points lower compared to a no-crisis counterfactual. Only by using a general equilibrium frame it is possible to disentangle permanent effects on productivity from the stationary fluctuations in output and determine the sources of the persistent productivity loss.

The quantitative section also studies the importance of the key mechanisms - heterogeneity, selection, and firm dynamics- in shaping the cost of financial crises. To that end, we calibrate alternative models that lack these features and study the welfare cost of a 100-basis-points mean-reverting increase in the interest rate. Comparing the baseline model with one that features an exogenous path for productivity, and therefore entails no productivity loss, we conclude that 27% of the consumption equivalent welfare cost in the baseline model is due to the permanent productivity loss. When the baseline is compared to a model that features endogenous productivity but lacks heterogeneity and firm dynamics, the consumption equivalent welfare cost is overestimated by almost 60%. This difference is a large economic magnitude that can bias public policy. Finally, we conclude our analysis by presenting new empirical evidence that corroborates the key implication of the model: industries with less heterogeneity should see larger decreases in entry and productivity

[4] In Ates and Saffie (2013), we study how firm selection at entry can reconcile the strongly detrimental effect of corporate taxes on firm entry with their milder and nonlinear effect on economic growth. That paper compares balanced growth path of a continuous-time closed-economy setting without aggregate risk arising from permanent differences in tax policy.

during the sudden stop. We revisit our empirical analysis on cross-industry heterogeneity to show that, indeed, industries that experienced higher entry rates gave rise to fewer productive firms during the sudden-stop episode.

The structure of the paper is as follows. Section 2 reviews the related literature. Section 3 introduces our model. Section 4 presents the analysis of the Chilean economy as a *pseudo* natural experiment for the model, exploring at the macro and micro levels the consequences of the sudden stop for the Chilean economy. Section 5 presents the calibration of the model and the quantification of the long-run cost of the sudden stop. Finally, Section 6 concludes and suggests avenues for future research.

2. Related Literature

The literature has documented the lack of a neoclassical recovery after large down turns (Cerra and Saxena, 2008; Reinhart and Rogoff, 2014). In particular, large economic downturns are characterized by economic hysteresis, with output and investment exhibiting permanent losses in levels, accompanied also by permanent productivity losses (Meza and Quintin, 2007; Blyde et al., 2010). To explain the permanent effects (hysteresis on output), the quantitative literature resorted to models that combine endogenous growth with dynamic stochastic general equilibrium (DSGE) framework. Originally, this mixture was used to amplify the welfare cost of business cycles (Barlevy, 2002), to generate low frequency business cycles (Comin and Gertler, 2006), and to study asset-price dynamics (Kung and Schmid, 2015). Most of this literature models endogenous growth based on Romer (1990). In this setup, all productivity growth comes from the introduction of new goods, and every new good has the same contribution to aggregate productivity growth. Therefore, in the event of a crisis, the entry of new goods decreases and the economy is permanently scarred (Queraltó, 2013; Gornemann, 2014; Guerrón-Quintana and Jinnai, 2014).

Our analysis contributes to the quantitative literature on output hysteresis by incorporating both firm dynamics and firm heterogeneity in addition to firm entry. The empirical literature on firm-level productivity has shown that the contribution of firms to aggregate productivity is both dynamic and heterogeneous (Bartelsman et al., 2009; Decker et al., 2016; Akcigit and Kerr, 2018). Our framework reflects these empirical findings, establishing the quantitative importance of these margins in evaluating the productivity cost of crises. To incorporate these features, we deviate from the Romer (1990) framework and build on the

Schumpeterian creative destruction framework of Klette and Kortum (2004).[5] Endogenous growth models based on Klette and Kortum (2004), where firms expand and contract, are successful in replicating the size distribution and other salient features of the micro data (Lentz and Mortensen, 2008). We also contribute to this literature by providing a novel mapping of the continuous time endogenous productivity models with firm dynamics and heterogeneity into discrete time stochastic general equilibrium models. In fact, this is the first paper that allows for aggregate risk in the Klette and Kortum (2004) framework. Moreover, our quantitative analysis shows that the firm-size distribution provides critical information when quantifying the hysteresis of financial crises. The framework can be easily embedded into large-scale DSGE models, making it suitable for estimation and policy.

A related literature has developed models of firm dynamics with aggregate risk that can generate an amplification of business cycle fluctuations (Bilbiie et al., 2012; Jaimovich and Floetotto, 2008) and selection effects (Khan and Thomas, 2013; Lee and Mukoyama, 2015; Clementi and Palazzo, 2016) consistent with our firm-level evidence. Firm heterogeneity and selection are also key channels in studies of the effects of international crises on aggregate productivity (Choi, 2012; Gopinath and Neiman, 2014).[6] This work highlights that deviating from perfect competition and representative firm frameworks helps rationalize the first-order effect of financial crises on aggregate productivity (cf. Kehoe and Ruhl, 2009). In this line of studies, productivity is exogenous to firm decisions. Therefore, while these frameworks can generate persistent effects, they are inherently not well equipped for analyzing permanent economic effects of stationary economic downturns. Similarly, models of misallocation (Buera et al., 2011; Buera and Shin, 2013; Midrigan and Xu, 2014) could generate slow transitions between steady states (Moll, 2014), but ultimately only permanent changes in fundamentals give rise to permanent changes in measured productivity in the long run. Therefore, in contrast to this literature, this paper develops a framework that incorporates endogenous productivity dynamics in business cycle analysis to study permanent productivity losses

[5]A detailed review of this literature can be found in Aghion et al. (2014). Klette and Kortum (2004) extends the pioneering work of Grossman and Helpman (1991) and Aghion and Howitt (1992) to incorporate horizontal expansion of incumbent firms. Applications of this framework include Acemoglu et al. (2018), Akcigit et al. (2013), Akcigit et al. (2016), Akcigit and Kerr (2018), Garcia-Macia et al. (2016), and Lentz and Mortensen (2008, 2016) among others.

[6]Gopinath and Neiman (2014) show that an increase in the price of imported inputs can trigger temporary productivity losses in the economy. Choi (2012) shows that fluctuations in firm entry rate and subsequent shifts in establishment composition toward smaller sizes following a sudden stop can explain the under-performance of TFP relative to the pre-crisis period in Korea during the Asian crisis. Both papers study perfect foresight economies, where crises and transitions are triggered by a one time unexpected change in a model fundamental.

stemming from stationary fluctuations, providing an empirically consistent micro-foundation for models with trend shocks (Aguiar and Gopinath, 2007) and disaster risk (Gourio, 2012).

On empirical grounds, this paper provides novel evidence using Chilean firm-level data and shows that financial crises generate permanent imprints on characteristics of newly founded firms. Thus, this paper is also related to the empirical literature that uses firm-level data to study financial crises (Hallward-Driemeier and Rijkers, 2013; Schnabl, 2012; Siemer, 2014; Sedláček and Sterk, 2017; de Ridder, 2017; Duval et al., 2017). In particular, de Ridder (2017) and Duval et al. (2017) provide micro evidence on firms' innovation decisions during financial crises. Both studies document significant damping effects of the Global Financial Crisis on firms' productivity enhancement activities. These empirical findings lend support to our modeling set-up where productivity is endogenously affected by the financial crisis.[7]

3. A Stochastic Small Open Economy

In this section, we present a small open economy model that is augmented with endogenous technical change, firm heterogeneity, financial selection, and firm dynamics, subject to stationary interest rate and productivity shocks. The international finance structure of the model is similar to Neumeyer and Perri (2005) and the endogenous technical aspect is inspired by Klette and Kortum (2004). There are four major economic agents in the economy. First, the final good producer combines capital and non-tradable intermediate products to produce the unique tradable good in the model. The production of the final good is subject to the aggregate productivity shock. Second, there is a fixed mass of intermediate products dominated by incumbent firms, and incumbents can own several differentiated intermediate products. Incumbents increase their number of intermediate products by optimally choosing their expansion effort. Third, a representative financial intermediary buys a portfolio of business ideas from the household and funds the most promising ones. Funded business ideas become entrants; i.e., new firms that start producing a single and differentiated intermediate product. Therefore, the endogenous productivity in this economy increases as a

[7]For the United States, de Ridder (2017) argues firms that relied on loans from banks whose balance sheet were severely affected by the crisis cut back research and development (R&D) investment relatively more, which ultimately weighed on their productivity growth. Duval et al. (2017) shows that this effect is also present also in other countries. In our model, a financial crisis affects productivity because higher interest rates decrease the incentives for firms' innovation. This is consistent with the negative association between monetary policy shocks, credit extension to firms and their R&D expenditure (see Abuka et al., 2019 and Jiménez et al., 2014 for evidence on credit channel in micro data; and Moran and Queraltó, 2018 for a VAR analysis of aggregate data).

consequence of the expansion of incumbents and the entry of new firms. Fourth, the representative household consumes the final tradable good and provides labor and capital services to the economy. She trades bonds and goods with the rest of the world. The interest rate she faces on the bond follows an exogenous stochastic process. The links between these agents are illustrated in Figure X in Appendix 1.1. We will first introduce the fundamentals of the model and then characterize the equilibrium relationships.

3.1. Final Good Producer

Time is discrete. We denote a history $(s_0, s_1, ..., s_t)$ by s^t, where s^t contains all the relevant past information that agents need to make decisions in period t. There is a representative final good producer that combines intermediate products $(\{X_j(s^t)\}_{j\in[0,\Lambda]})$, indexed by $j \in [0, \Lambda]$, with capital $(K(s^t))$, to produce the only final good of this economy $(Y(s^t))$. The parameter $\Lambda > 0$ determines the mass of intermediate products in the economy and is time invariant. The constant return to scale production function is given by

$$\ln Y(s^t) = z(s^t) + \frac{\alpha}{\Lambda}\int_0^\Lambda \ln X_j(s^t)dj + (1-\alpha)\ln K(s^t), \tag{1}$$

where $z(s^t)$ is the exogenous component of aggregate productivity and is characterized by the following AR(1) process:

$$z(s^t) = \rho_z z(s^{t-1}) + \sigma_z \epsilon_z(s^t) \quad \epsilon_z(s^t) \overset{iid}{\sim} N(0,1). \tag{2}$$

Equation (1) is an extension of a standard unit elastic production function where α determines the production share of the mass Λ of intermediate products.

3.2. Intermediate Good Producers

3.2.1. Production

In each product line, two firms compete à la Bertrand for the ownership of production. Each intermediate product is owned by the firm that can produce it with higher productivity, and a firm can own several intermediate products. Firms are indexed by f, and the measure of firms is denoted by $\Omega(s^t) \in (0, \Lambda]$, which is an endogenous object. A firm is defined by a collection of products $\mathbb{J}_f = \{j : j \text{ is owned by firm } f\}$. Production technology requires labor

$(l_j(s^t))$ as input. The production of intermediate product j is given by

$$X_j(s^t) = l_j(s^t)q_j(s^t), \tag{3}$$

with $q_j(s^t)$ denoting the efficiency of labor.

The efficiency of labor $(q_j(s^t))$ evolves with each technological improvement generated by successful innovations. Innovations are heterogeneous in their capacity to improve the existing technology. Drastic innovations are generated by high-type firms and they improve the efficiency level by a factor of $1+\sigma^H$, while marginal innovations, performed by low-type firms, generate improvements with a smaller factor $1+\sigma^L$. Firm types are determined at the entry stage and remain fixed thereafter. We can define the indicator functions $I_j^d(s^{t-1}, s_t)$ taking the value 1 if product line j receives an innovation of type $d \in \{L, H\}$ under $s^t = (s^{t-1}, s_t)$ and 0 otherwise. We can summarize the evolution of the productivity in product line j as follows:

$$q_j(s^t) = \left(1 + I_j^H(s^{t-1}, s_t) \cdot \sigma^H + I_j^L(s^{t-1}, s_t) \cdot \sigma^L\right) \cdot q_j(s^{t-1}). \tag{4}$$

Hence, productivity in intermediate product j remains unchanged next period if and only if no innovation takes place in j. In this case, the same firm continues producing that intermediate product.

Another feature of the production process is that labor input is subject to a working capital constraint as in Neumeyer and Perri (2005). In particular, the intermediate producer needs to hold a proportion $\eta > 0$ of the wage bill before production takes place.[8] To do so, she borrows at the interest rate at the beginning of the period and pays back just after production takes place.

3.2.2. Extensive Product Margin: Expansion and Contraction of Firms

In this economy, productivity enhancements arise endogenously as a result of both the expansion decision of incumbents and the entry decision of firms. Therefore, the endogenous evolution of the productivity of each intermediate product is determined by the firm dynamics of the economy. Figure XII in Appendix 1.2 illustrates how the expansion of incumbents and the entry of new firms govern both the number of intermediate products owned by a

[8] This feature has no quantitative impact on the measurement of the long-run cost of a financial crisis, but it improves the business cycle properties of the economy. See Online Appendix 3.2.1 for details.

firm and the evolution of the productivity of each product.

A d-type firm owning n product lines can engage in innovation to acquire technological leadership over other intermediate products by hiring labor. In particular, a d-type firm with n product lines hires $l_r^d(s^t)$ workers per product for expansion purposes.[9] We assume that a firm acquires new product lines according to a binomial process with success probability $\iota^d(s^t)$ and n trials, where $\iota^d(s^t)$ is given by

$$\iota^d(s^t) = \left(\frac{l_r^d(s^t)}{\varphi}\right)^{\frac{1}{\xi}}, \quad \text{where} \quad \xi > 1 \quad \text{and} \quad \varphi > 0. \tag{5}$$

This setup suggests that ideas for improving upon new products come from the existing products dominated by the firm. For every product that a firm owns, a potential new application or a spinoff product arises with probability $\iota^d(s^t)$. When a firm with n product lines hires $l_r^d(s^t)$ workers per product line, it generates in expectation $\mathbb{N}^d \equiv n \cdot \iota^d(s^t)$ new products. Therefore,

$$\mathbb{N}^d = \left(\frac{1}{\varphi}\right)^{\frac{1}{\xi}} \left(l_r^d(s^t)\right)^{\frac{1}{\xi}} n. \tag{6}$$

Reciprocally, the cost of generating in expectation $\mathbb{N}$ new products for a firm with n products is given by

$$cost(\mathbb{N}) = \varphi \frac{\bar{W}(s^t)\left(\mathbb{N}^d\right)^{\xi}}{n^{\xi-1}} \quad \text{for} \quad \mathbb{N} \leq n. \tag{7}$$

Intuitively, the more product lines a firm has, the cheaper it is to acquire new products, and the higher the wage or the interest rate in the economy is, the more costly it is to acquire new products.[10]

Our framework for firm dynamics builds on Klette and Kortum (2004). In their continuous time model with Poisson arrival rates, multiple acquisitions or multiple losses of products have negligible probability at any instant.[11] However, multiple acquisitions cannot be avoided when working in discrete time, as we explain in more detail later. Therefore, this paper proposes a discrete time mapping of the continuous time endogenous technical change literature with firm dynamics to handle multiple acquisitions of products. This mapping is

[9] Anticipating that, in equilibrium, labor employed and innovation intensity per line are the same across individual lines of a firm, we drop the subscript j describing these variables.

[10] The expansion cost function described in equation (7) is isomorphic to the one proposed by Klette and Kortum (2004). The authors use this structure to match one of their stylized facts, which states that the firm growth rate decreases in size (conditional on survival), while size and growth rates are uncorrelated among large firms (Gibrat's Law), in line with empirical surveys (Sutton, 1997).

[11] An application of Itô's Lemma.

the key to introducing firm dynamics and endogenous technical change into a fully stochastic business cycle model in a tractable way.

3.3. Financial Intermediary and Selection: Entry

The entry of new firms is determined by the funding of business ideas. A continuum of identical risk-neutral financial intermediaries buy from the household a unit mass of heterogeneous business ideas every period. Because of perfect competition between financial intermediaries, the price of the portfolio is given by the expected profits arising from managing it. The portfolio is a continuum of business ideas indexed by h and uniformly spread on the unit interval ($h \in [0, 1]$). The fixed cost of starting a business idea is κ units of labor. Conditional on paying the fixed cost, a project generates a new firm with one product. Business ideas are heterogeneous in their expected step size; every project has an idiosyncratic probability $P^H(h) = h^\nu$ ($\nu > 0$) of generating a high-type firm characterized by step size $\sigma^H > \sigma^L$. The higher the index h, the more likely that project h will generate a high-type firm.[12] In this regard, h is more than an index; it is a ranking among business ideas. The parameter ν governs the scarcity of good ideas in this economy. Therefore, the implied probability distribution of P^H is given by

$$f(P^H) = \frac{1}{\nu}\left(\frac{1}{P^H}\right)^{1-\frac{1}{\nu}}.$$

The mean of this distribution is given by $\underline{\tilde{\mu}} = \frac{1}{\nu+1}$, and reflects the expected proportion of high-type entrants when a set of business ideas are funded randomly. The skewness of $f(P^H)$ is $Sk(\nu) = \frac{2(\nu-1)\sqrt{1+2\nu}}{1+3\nu}$. Therefore, ν summarizes the underlying scarcity of promising ideas in the economy.

3.4. The Representative Household

In line with the small open economy business cycle literature, there is a representative consumer that trades bonds in international bond markets, rents capital for production, and is subject to both capital and bond adjustment costs. The household derives utility from

[12] Heterogeneity and scarcity of ideas is one explanation for the high skewness in firm-level variables. See, for instance, Scherer (1998) and Silverberg and Verspagen (2007).

consumption and leisure, admitting the following life-time utility:

$$U(t) = \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[u(C(s^t), l(s^t))|s_0\right] \tag{8}$$

where $C(s^t)$ and $l(s^t)$ denote state-contingent sequences of consumption and labor, respectively; $E[\bullet|s_0]$ is the expectation over history s^t, conditional on the information at $t = 0$; and $0 < \beta < 1$ is the constant discount factor. Let us similarly denote sequences of bond holding by $B(s^t)$ and of investment by $I(s^t)$. Given sequences of interest rates $R(s^t)$, wages $W(s^t)$, capital rental rates $r(s^t)$, together with initial bond and capital holdings, the budget constraint of the household is defined as

$$\begin{aligned} C(s^t) \leq\ & W(s^t)l(s^t) + r(s^t)K(s^{t-1}) + B(s^{t-1})R(s^{t-1}) + T(s^t) - K(s^t) \\ & + (1-\delta)K(s^{t-1}) - \Phi(\bullet) - B(s^t) - \Psi(\bullet), \end{aligned} \tag{9}$$

where investment is subject to convex adjustment costs $\Phi(\bullet)$, and bond holdings are subject to the convex cost function $\Psi(\bullet)$. In every period the household receives a lump sum transfer $T(s^t)$ from the ownership of the representative financial intermediary and the firms. In this small open economy, the interest rate is exogenous, and we use the following AR(1) process to model it:

$$\ln\left(\frac{R(s^t)}{\bar{R}}\right) = \rho_R \ln\left(\frac{R(s^{t-1})}{\bar{R}}\right) + \sigma_R \epsilon_{R,t} \quad \text{where} \quad \epsilon_{R,t} \overset{iid}{\sim} N(0,1), \tag{10}$$

where $\bar{R}$ is the long-run interest rate in the economy. The problem also requires transversality conditions on capital and bond holdings.

We modify Greenwood et al. (1988) preferences (GHH) to allow for a balanced growth path equilibrium. Because aggregate labor productivity ($A(s^t)$) grows at an endogenous rate, the scaling of labor disutility is time variant. The functional forms for are given by:

$$u(C(s^t), l(s^t)) = \frac{1}{1-\gamma}\left(C(s^t) - \Theta A(s^t)\left(l(s^t)\right)^{\chi}\right)^{1-\gamma} \tag{11}$$

$$\Psi(B(s^t), Y(s^t)) = \frac{\psi}{2} Y(s^t)\left(\frac{B(s^t)}{Y(s^t)} - \bar{b}\,(1+\bar{g})\right)^2 \tag{12}$$

$$\Phi(K(s^{t-1}), K(s^t)) = \frac{\phi}{2} K(s^{t-1})\left[\frac{K(s^t)}{K(s^{t-1})} - (1+\bar{g})\right]^2. \tag{13}$$

where $\Theta > 0$ is the labor dis-utility level, $\chi > 1$ determines the Frisch elasticity of labor supply, $\left(\frac{1}{\chi-1}\right)$, γ is the inverse of the intertemporal elasticity of substitution, and $\phi > 0$ and $\psi > 0$ determine the convexity of the cost functions. Because $\bar{b}$ is the long-run household debt-output ratio and $\bar{g}$ is the long-run growth of the economy, the household pays neither adjustment nor bond holding costs along the balanced growth path.

3.5. Equilibrium

In this section, we characterize the equilibrium relationships of the model economy. We provide a description of the timing of events in the economy in Appendix 1.1 and, for the sake of brevity, defer the exact definitions of the equilibrium and the balanced growth path in this economy to Appendix 1.3.

3.5.1. Households

We start describing the equilibrium relationships with the representative household. The household chooses state-contingent sequences of consumption $C(s^t)$, labor $l(s^t)$, bond holding $B(s^t)$, and investment $I(s^t)$, given sequences of prices $\{W(s^t), R(s^t), r(s^t)\}$ and initial bond and capital positions, solving the following:

$$\max_{\{B(s^t),C(s^t),l(s^t),I(s^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[u(C(s^t), l(s^t))|s_0\right] \tag{14}$$

subject to the budget constraint given in equation (9). The stochastic discount factor of the household ($m(s^t, s_{t+1})$) follows as

$$m(s^t, s_{t+1}) = \beta \frac{\dfrac{\partial u(C(s^t, s_{t+1}), l(s^t, s_{t+1}))}{\partial C(s^t, s_{t+1})}}{\dfrac{\partial u(C(s^t), l(s^t))}{\partial C(s^t)}}.$$

Because households are the ultimate owners of the firms, this stochastic discount factor also characterizes the value of the firm.

3.5.2. Final Good Producer

Given input prices ($p_j(s^t)$) and the rental rate of capital ($r(s^t)$), the final good producer demands intermediate products and capital in every period in order to solve

$$\max_{\{X_j(s^t)\}_{j\in[0,\Lambda]},K(s^t)} \left\{ Y(s^t) - \int_0^\Lambda X_j(s^t)p_j(s^t)dj - K(s^t)r(s^t) \right\}. \tag{15}$$

The solution to final good producer's problem is characterized by the following set of demands for intermediate products and capital:

$$X_j(s^t) = \frac{\frac{\alpha}{\Lambda}Y(s^t)}{p_j(s^t)} \quad \forall j \ , \tag{16}$$

$$K(s^t) = \frac{(1-\alpha)Y(s^t)}{r(s^t)}. \tag{17}$$

Because of the unit elastic demand, a monopolist of intermediate product j facing the demand in equation (16) would choose $p_j(s^t) \to \infty$ and, hence, $X_j(s^t) \to 0$. Only the existence of a potential competitor can force the intermediate producer to set a finite price in a given product line. The next subsection introduces Bertrand monopolistic competition within each intermediate product, providing a rationale for limit pricing.

3.5.3. Intermediate Good Producer

Denote the endogenous replacement probability of this economy by $\Delta(s^t)$. Because replacement is undirected, every product line faces the same type-independent probability $\Delta(s^t)$ of receiving an innovation and being dominated by a new firm starting next period. Define $\mathbb{P}(k, n, p)$ as the probability of observing k events in a binomial process with n trials and success probability p:

$$\mathbb{P}\left(k, n, p\right) = \binom{n}{k} \left(p\right)^k \left(1-p\right)^{n-k}.$$

Now, the value of a d-type firm that currently owns n products can be written as

$$V^d(s^t,n) = \max_{\{\{p_j(s^t)\}_{j\in\mathbb{J}_f},\iota^d(s^t)\}} \left\{ \sum_{\mathbb{J}_f} \left[p_j(s^t)X_j(s^t) - \bar{W}(s^t)\frac{X_j(s^t)}{q_j(s^t)} \right] - n\bar{W}(s^t)\cdot\varphi\cdot\iota^d(s^t)^{\xi} + \right. \tag{18}$$

$$\left. \mathbb{E}\left[m(s^t,s_{t+1})\cdot \left(\sum_{\tilde{k}=0}^{n} \mathbb{P}\left(\tilde{k},n,\Delta(s^t)\right) \sum_{k=0}^{n} \left[\mathbb{P}\left(k,n,\iota^d(s^t)\right) V^d(s^{t+1},n-\tilde{k}+k) \right] \right) |s^t \right] \right\},$$

subject to

$$X_j(s^t) = \frac{\frac{\alpha}{\Lambda}Y(s^t)}{p_j(s^t)} \text{ if } p_j(s^t) \le \tilde{p}_j(s^t) \text{ and zero otherwise,}$$

where $\tilde{p}_j(s^t)$ denotes the competitor's price, and $\bar{W}(s^t)$ denotes the wage bill inclusive of working capital constraints (we will detail this below).[13] Here, $\mathbb{E}[\bullet|s^t]$ denotes the conditional expectation over every possible s_{t+1}, event after history s^t and $m(s^t,s_{t+1})$ is the stochastic discount factor of the representative household. When writing this expression, we used the conjecture that the firm value only depends on the firm type. As shown below, this holds true as production and innovation decisions are independent of product-line specific characteristics, with the firm type being a sufficient statistic, given the aggregate state.

Taking wages and the demand from the final good producer as given, the firm optimally chooses the price level to charge for each intermediate good it produces subject to its competitor's price, which is reflected by the constraint. It also decides on the innovation rate per product line. The first line in equation (18) captures the profits it generates from its product lines at time t and the total expenditure on R&D employment. The second component reflects the expectation over all potential product portfolios at time $t+1$ given the aggregate replacement rate and the endogenous expansion decision. Because there is a continuum of firms, each firm takes the replacement process as given. Therefore, the binomial process governing innovation and the binomial process governing the destruction of product lines are independent when characterizing the expected value.

First, we describe the static decisions of the firm. Bertrand monopolistic competition in intermediate product markets implies that the competitor with the lower marginal cost dominates the market by following a limit pricing rule—i.e., she sets her price ($p_j(s^t)$) equal to the marginal cost of the closest follower. We can denote the efficiency of the closest

[13] For brevity, we do not present the static price-setting problem of the follower, which is defined reciprocally. As the follower has a disadvantage in terms of marginal production cost, the optimal price the leader charges becomes the marginal cost of the follower in equilibrium. As such, the follower has no incentive to deviate and does not produce in equilibrium.

follower by $\tilde{q}_j(s^t)$. Then we have

$$p_j(s^t) = \frac{W(s^t)}{\tilde{q}_j(s^t)} \left(1 + \underbrace{\eta(R(s^{t-1}) - 1)}_{\text{Cost wedge}} \right) = \frac{\bar{W}(s^t)}{\tilde{q}_j(s^t)}. \tag{19}$$

The firm uses intra-period loans to find its working capital needs and, therefore, pays the interest rate of the previous period on the loans. To simplify notation, we define the adjusted wage as $\bar{W} = (1 + \eta(R(s^{t-1}) - 1))\, W(s^t)$. Equation (4) implies that a d-type leader has productivity $q_j(s^t) = (1 + \sigma^d) \cdot \tilde{q}_j(s^t)$. Then, using the demand for intermediate products of the final good producer from (16), we derive the following expression for the profits of the leader in product line j with productivity advantage d:

$$\Pi_j^d(s^t) = X_j(s^t) \left(p_j(s^t) - \frac{\bar{W}(s^t)}{q_j(s^t)} \right) = \frac{\alpha Y(s^t)}{\Lambda} \frac{\sigma^d}{(1 + \sigma^d)}. \tag{20}$$

Conveniently, profits only depend on the type of the current leader and the state of the economy, not on the productivity level of the product ($q_j(s^t)$). Moreover, high-type leaders always enjoy higher profits per product line than low-type ones do. The labor employed in the production of each intermediate product is also independent of the productivity level:

$$l_j^d(s^t) = \frac{\alpha Y(s^t)}{\Lambda \bar{W}(s^t)\, (1 + \sigma^d)}. \tag{21}$$

Having characterized the pricing and production decision of incumbent firms, we turn to their optimal expansion decisions. To solve for the innovation rate and the value function, we guess and verify that

$$V^d(s^t, n) = n \cdot \bar{V}^d(s^t),$$

where $\bar{V}(s^t)$ are independent of n. Imposing the former guess in equation (18) we can solve for the optimal innovation rate:

$$\iota^d(s^t) = \left(\frac{\mathbb{E}\left[m(s^t, s_{t+1}) \bar{V}^d(s^{t+1}) | s^t \right]}{\varphi \xi \bar{W}(s^t)} \right)^{\frac{1}{\xi - 1}}. \tag{22}$$

The optimal innovation rate is independent of the number of products. Intuitively, innovation intensity is increasing in the value of new products, decreasing in the effective wage, and a larger discount factor also decreases innovation incentives. The optimal number of research workers per product line is given by:

$$l_r^d(s^t) = \varphi \iota^d(s^t)^{\xi}. \tag{23}$$

Thus, the value of a single product line for a d-type firm is recursively defined as

$$\begin{aligned}\bar{V}^d(s^t) &= \Pi^d(s^t) - \bar{W}(s^t)\varphi\iota^d(s^t)^{\xi} \\ &\quad + \mathbb{E}\left[m(s^t, s_{t+1})\left[\left(1 - \Delta(s^t) + \iota^d(s^t)\right)\bar{V}^d(s^{t+1})\right]|s^t\right].\end{aligned} \tag{24}$$

The resulting value of a firm is indeed linear on the number of products in its portfolio, confirming the original guess. Here, $\bar{V}^d(s^t)$ is the value of a type d firm with one product line and $\bar{V}^H(s^t) > \bar{V}^L(s^t)$ implying that high-type firms expand at a higher rate than low-type firms do. Conveniently, *ex-post* firm heterogeneity can be summarized by $d \in \{L, H\}$ because every type d firm charges the same markup, hires the same number of workers, earns the same profits, and innovates at the same rate per product line. Therefore, there is no need to keep track of the distribution of labor productivity across product lines.

3.5.4. Entry

Because business ideas are heterogeneous and good ideas are scarce, selection plays a critical role in this economy. The representative financial intermediary performs that task. In particular, it borrows funds to cover working capital needs and selects business ideas to fund according to their expected present value. Because $\bar{V}^H(s^t) > \bar{V}^L(s^t)$, the representative financial intermediary strictly prefers to fund business ideas with higher h. Therefore, the optimal strategy for a financial intermediary financing $M(s^t)$ business ideas at time t is to set a cutoff $\bar{h}(s^t) = 1 - M(s^t)$ and to fund business ideas only with $h \geq \bar{h}(s^t)$.[14] When the financial intermediary selects a mass $M(s^t)$ of business ideas, the proportion $\tilde{\mu}(\bar{h}(s^t))$ of high-type firms in the funded business ideas is given by

$$\tilde{\mu}(M(s^t)) = \frac{1}{M(s^t)}\int_{1-M(s^t)}^{1} P^H(h)\,dh = \underbrace{\frac{1}{\nu+1}}_{\underline{\tilde{\mu}}} \times \underbrace{\left[\frac{1 - [1 - M(s^t)]^{\nu+1}}{M(s^t)}\right]}_{\geq 1}. \tag{25}$$

[14]Because the expected value is strictly increasing in the idiosyncratic probability of becoming a high-type firm and the funding cost is fixed, the cutoff strategy is optimal and unique.

For any mass ($M(s^t)$), the fraction of high-type entrants ($\tilde{\mu}(M(s^t))$) decreases with the scarcity of high-type business ideas (ν). Moreover, in terms of the resulting composition, financial selection performs at least as well as random selection.

Given $\left\{\bar{V}^H(s^t),\, \bar{V}^L(s^t),\, R(s^{t-1}))\right\}$, the representative financial intermediary chooses $M(s^t)$ in order to solve

$$\max_{M(s^t)\in(0,1)} \left\{ \underbrace{M(s^t)}_{\text{Cohort's mass}} \left[\underbrace{\mathbb{E}\left[m(s^t, s_{t+1}) \left\{\tilde{\mu}(M(s^t))\bar{V}^H(s^{t+1}) + (1-\tilde{\mu}(M(s^t))\bar{V}^L(s^{t+1})\right\} |s^t\right]}_{\text{Cohort's expected value}} \right] \right.$$
$$\left. - \underbrace{M(s^t)\bar{W}(s^t)\kappa}_{\text{Total cost of funding}} \right\}. \tag{26}$$

The bracketed term is the expected return of the portfolio with composition $\tilde{\mu}\left(M(s^t)\right)$. As equation (25) shows, the financial intermediary faces a tradeoff between mass and composition of the funded pool: A higher $M(s^t)$ increases the mass of new firms, but it also decreases the average value of the entrant cohort. If an interior solution (i.e., $M(s^t) \in (0,1)$) exists, it is unique and characterized by

$$M(s^t) = 1 - \left(\frac{\bar{W}(s^t)\kappa - \mathbb{E}\left[m(s^t, s_{t+1})\bar{V}^L(s^{t+1})|s^t\right]}{\mathbb{E}\left[m(s^t, s_{t+1}))\left(\bar{V}^H(s^{t+1}) - \bar{V}^L(s^{t+1})\right)|s^t\right]} \right)^{\frac{1}{\nu}}. \tag{27}$$

The optimal mass ($M(s^t)$) decreases with the interest rate because of both the working capital constraint and the increase on the stochastic discount factor.[15] Therefore, a higher interest rate implies a smaller cohort with a higher fraction of high-type firms.

The next subsection derives the main equation that links incumbent and entrant innovation to aggregate productivity.

[15]Interestingly, equation (27) can also be the outcome of a different setup where each entrepreneur seeks funding for business plans independently. Under that decentralization, the marginal entrepreneur would be indifferent between borrowing and starting a firm or not starting her project:

$$\bar{W}(s^t)\kappa = \mathbb{E}\left[m(s^t, s_{t+1})\left(\bar{h}(s^t)^{\nu}\bar{V}^H(s^{t+1}) + (1-\bar{h}(s^t)^{\nu})\bar{V}^L(s^{t+1})\right)|s^t\right]. \tag{28}$$

Simple algebra shows that equation (28) and equation (27) are the same mathematical object. In this sense, the model is silent about the nature of the financial selection process. Selection can arise because the best entrepreneurs apply for funding or because the financial intermediary rejects bad applicants. Only direct data on application and rejection of credit can tell these stories apart. For instance, the second explanation implies large variations of the rejection to application ratio during crises, while the first one does not. Although these data are not available for Chile, Jiménez et al. (2014)'s loan-level data for Spain are consistent with large variations in the rejection to application ratio.

3.6. Endogenous Replacement and Productivity Growth

To close the model, we now characterize the endogenous replacement of products ($\Delta(s^t)$) and the fraction of products controlled by high-type firms ($\mu(s^t)$), which determine the endogenous productivity growth of the economy. The variables $\iota^H(s^t)$, $\iota^L(s^t)$, and $M(s^t)$ are determined in period t but materialize at the beginning of period $t+1$. Because the product space is continuous and because replacements by entrants and incumbents occur simultaneously, the same product line cannot be acquired by two firms in the same period and the probability of an incumbent innovating on its own product is zero. Therefore, we can define the aggregate replacement rate of the economy as

$$\Delta(s^t) \equiv \underbrace{\frac{M(s^t)}{\Lambda}}_{\text{Replacement by Entrants}} + \underbrace{\mu(s^t)\iota^H(s^t) + \left(1-\mu(s^t)\right)\iota^L(s^t)}_{\text{Replacement by Incumbents}}. \tag{29}$$

We can also derive the law of motion for the composition of product lines:

$$\mu(s^{t+1}) = \mu(s^t) + \underbrace{\frac{M(s^t)}{\Lambda}\left[\tilde{\mu}(M(s^t)) - \mu(s^t)\right]}_{\text{Entrants}} + \underbrace{\mu(s^t)\left(1-\mu(s^t)\right)\left(\iota^H(s^t) - \iota^L(s^t)\right)}_{\text{Incumbents}}. \tag{30}$$

A higher fraction of high types in the entrant cohort ($\tilde{\mu}(M(s^t))$) implies a higher fraction of product lines dominated by high-type incumbents. Also, larger gaps between the innovation rate of high and low types ($\iota^H(s^t) - \iota^L(s^t)$) trigger increases in the fraction of products dominated by high-type firms.

We can now derive an expression for the endogenous productivity process of this economy. Combining the production function from equation (1) and equation (3), and recognizing that intermediate labor used in the production of an intermediate product depends only on the step size of the incumbent that controls it, we obtain:

$$Y(s^t) = \exp\left(z(s^t)\right)\left(\underbrace{A(s^t)}_{\text{Endogenous}}\right)^{\alpha}\left[\left(l^H(s^t)\right)^{\mu(s^t)}\left(l^L(s^t)\right)^{1-\mu(s^t)}\right]^{\alpha}\left(K(s^{t-1})\right)^{1-\alpha} \tag{31}$$

where $A(s^t)$ is defined as

$$\ln A(s^t) \equiv \frac{1}{\Lambda}\int_0^{\Lambda} \ln q_j(s^t)dj.$$

$A(s^t)$ is endogenous, and we can characterize it using the evolution of firm-level labor productivity in equation (4) together with the entry rate and the innovation decisions of incumbents. In particular, the growth rate of $A(s^t)$ is given by

$$1 + a(s_t) \equiv \frac{A(s^t, s_{t+1})}{A(s^t)} = \underbrace{\left[\left(1+\sigma^H\right)^{\tilde{\mu}(s^t)} \left(1+\sigma^L\right)^{1-\tilde{\mu}(s^t)}\right]^{\frac{M(s^t)}{\Lambda}}}_{\text{Entrants}}$$

$$\times \left[\underbrace{\left(1+\sigma^H\right)^{\mu(s^t)\iota^H(s^t)} \left(1+\sigma^L\right)^{\left(1-\mu(s^t)\right)\iota^L(s^t)}}_{\text{Incumbents}}\right]. \tag{32}$$

Equation (32) is at the core of the model; it shows that crises have permanent productivity effects because they affect the productivity accumulation process. Endogenous technical change provides a link between stationary fluctuations and productivity growth. The crisis affects productivity via the entry of new firms and innovation of incumbent firms. The contribution of entrants to productivity growth boils down to a scaled geometric weighted average of the step sizes, where the weights are given by the fraction of each type in the entrant cohort (composition) and the scale is given by the fraction of products that the cohort improves (mass). The innovation of incumbent firms is driven by two forces: First, equation (22) shows how fluctuations of the value of products during the crisis trigger changes in the expansion efforts of incumbents ($\iota^d(s^t)$). Second, because today's entrants are tomorrow's incumbents, changes in the composition of entrants have dynamic effects on the fraction of product lines dominated by high-type firms in future periods.[16]

The size distribution of firms and the productivity distribution are not needed to solve for the growth rate or any other macro aggregate. In particular, because the firm's decisions are i) independent of the specific productivity level and ii) scaled linearly on their number of product lines, the only distribution needed is the type composition of product lines ownership, perfectly defined by $\mu^d(s^t)$. Therefore, from a pure macro perspective, this model has only one additional state variable compared with a traditional small open economy real business cycle model. Nevertheless, from a firm-level perspective, the model has

[16] While the composition effect counteracts the adverse effects of a crisis on productivity, it is improbable that the composition effect dominates, leading to an increase in productivity. The reason is that there is a direct negative effect on incumbents' stochastic discount factor from the interest rate shock. That leads to a decline in incumbents' expansion incentives, even though the creative destruction, thus the probability of losing a product line, decreases with the entry rate decline triggered by higher interest rates. In Appendix 1.6, we elaborate further on the theory behind this result focusing on a positive interest rate shock—the driver of crises analyzed in this paper.

a well-defined size distribution that can be compared to micro data. Recovering the unique path of firm dynamics associated with the macro dynamics is critical in order to assess the firm-level performance of the model and use firm level moments for calibration. Appendix 1.2 characterizes the evolution of the size distribution. Before moving to the quantitative analysis, the next section provides empirical evidence supporting the mass and composition tradeoff that lies at the center of the model.

4. The Chilean Case: Fewer, But Better

In August 1998, the Russian sovereign default triggered a violent sudden stop in the developing world. Interest rate spreads for the seven biggest Latin American economies tripled in the weeks after this crisis. Figure I shows the effect of the crisis on the real interest rate and the current account-to-GDP ratio for Chile between 1995 and 2007. In 1998:Q3, the real interest rate was 225 basis points higher than a year earlier and reached pre-Asian crisis levels only in 2000:Q1. The sudden and persistent current account reversal lasted at least until 2002, drying out the Chilean capital markets.[17]

The goal of this section is to explore Chilean microeconomic data to assess empirically the main mechanism of the model, i.e., the existence of a mass-composition tradeoff on the entry margin. We start by showing the lack of recovery of the Chilean manufacturing sector from the sudden stop, then we use plant-level data to show that firms born in crisis are not just *fewer*, they are also *better*.

4.1. The Sudden Stop and the Missing Recovery

To document the persistent effects of crisis, the literature compares the post-crisis macroeconomic aggregates with an alternative path of steady growth at the pre-crisis average rate (e.g., see Cerra and Saxena 2008). Figure II repeats this exercise with real manufacturing GDP (log) to highlight the persistent consequences of the Chilean crisis.

Chilean real manufacturing GDP grew 6.2%, on average, between 1986–1997 with a standard deviation of 3%. The realized path after 1997 is substantially below the pre-crisis trend even eight years after the crisis, as shown in Figure II. Even under the conservative

[17] We use the average real interest rate for commercial loans with maturities up to three months reported by the Chilean Central Bank.

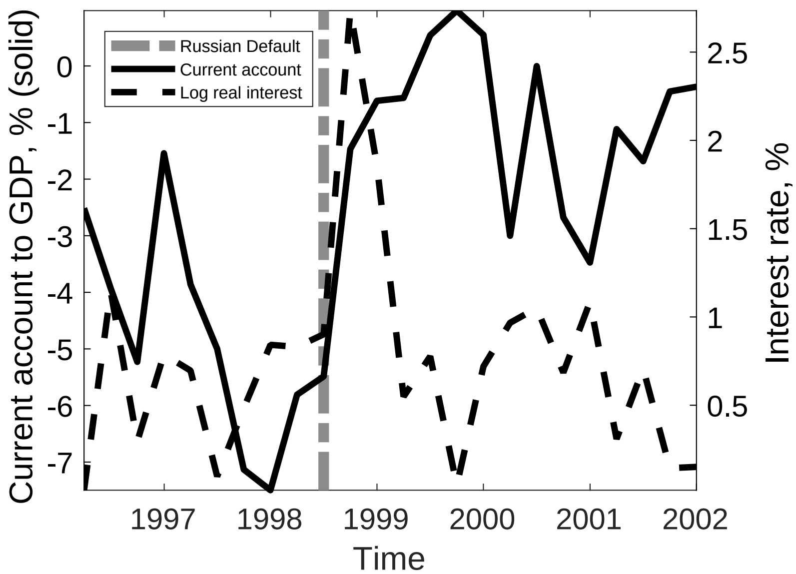


FIGURE I
Chilean Crisis: Real Interest Rate and Current Account-to-GDP ratio

Notes: The reported interest rate is the average real interest rate for commercial loans reported by the Chilean Central Bank for quarterly maturities. The current account data is retrieved from Haver Analytics database (based on Central Bank of Chile data) and is seasonally adjusted.

assumption that the trend growth has halved in the wake of the crisis, this post-crisis slowdown would imply that Chile's manufacturing output was 4.9% lower than the post-crisis trend on the eve of the Global Financial Crisis. This indicates a permanent and sizeable output loss associated with the financial crisis. Before making use of our structural framework to examine forces that generate this loss, we take advantage of the Chilean micro data to provide evidence of the mass and composition forces at the core of the model.

4.2. Mass and Composition during a Sudden Stop

There was no change in the domestic fundamentals of Chile that could have caused or predicted an increase in the interest rate as sudden and substantial as the one observed in the data. Chilean economy was growing robustly for a decade, its fiscal policy was steady and sober, and the monetary policy of its autonomous central bank was not expansionary. Moreover, as argued by Calvo et al. (2006), the generalized and synchronized nature of the increase in spreads charged in emerging markets also points to an exogenous and common origin for this episode. Thus, taking the Russian crisis as an exogenous shock, unrelated

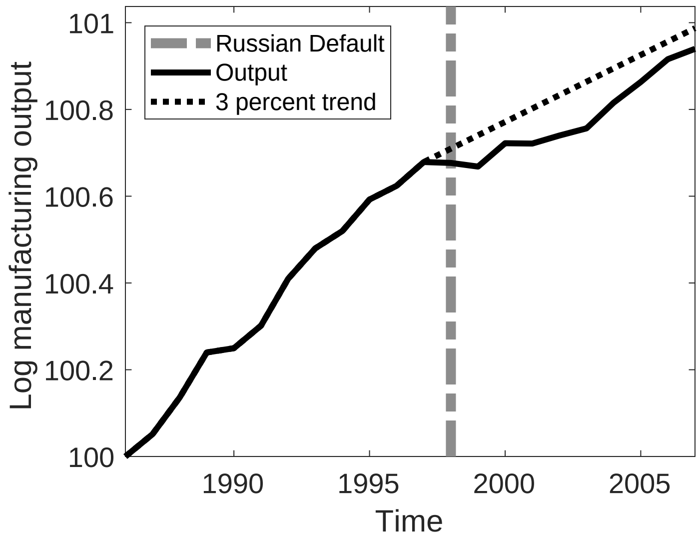


Figure II
Lack of Recovery in Manufacturing Output

Notes: The figure compares annual manufacturing real log GDP for Chile between 1986 and 2007 and an alternative path after the crisis with half the historical growth. On the eve of the great recession, Chile's manufacturing output is 4.9% below the alternative 3.1% trend.

to Chilean fundamentals, and completely unforeseen by firms and authorities, we perform a pseudo-natural experiment in order to test the main mechanism of the model: Cohorts born during the sudden stop window should be smaller but more productive.

Chile's National Institute of Statistics (INE) performs a manufacturing census (ENIA) every year, collecting plant-level data from every unit with more than 10 employees.[18] The survey contains yearly plant information on sales, costs, value added, number of workers, energy consumption, and other variables. For the empirical analysis, we use the information in the surveys between 1995 and 2007 to build a panel. We take the first appearance in the data as the entry year and the last appearance as the exit date.[19] The sample contains $9,446$ plants and $59,553$ observations.[20]

[18]Although firms can have multiple plants, we assume that every firm is a single plant. According to Pavcnik (2002) more than 90% of the Chilean manufacturing firms are single-plant firms.

[19]We include in the analysis 22 of the 29 three-digit industries. We exclude commodity-related industries (353 and 354 for petroleum, 371 and 372 for metals). We also drop industries where revenue productivity cannot be reliable estimated (one or the sum of the input elasticities are outside the unit circle, typically due to the lack of observations) this is the case for 361 (pottery), 323 (leather), and 314 (tobacco). After all the cleaning procedures, the sample has 85% of the firms-year observations and 90% of the workers. The most important drop is copper related (371 and 372), implying a combined loss of 2.3% of observations and 5.6% of workers). Online Appendix 2 shows the details of the data construction and a summary of the variables used in the analysis.

[20]Firms observed for the first time in 1995 are excluded from the regression analysis because their entry

We first calculate entry rates at year t at the industry level for each cohort, dividing the number of new plants in year t by the average of the total plants in years t and $t-1$. Figure III plots the average entry rates by industry for the crisis period (1998–2000) and the non-crisis years (1996–1997 and 2001–2006). Every industry below the 45^o line decreased its average entry rate during the crisis; the size of the markers reflects the share of workers of that industry in 1995. The weighted average decrease in entry is 2.6 percentage points, or a 29.3% decline from the weighted average in the pre-crisis years (8.6%).

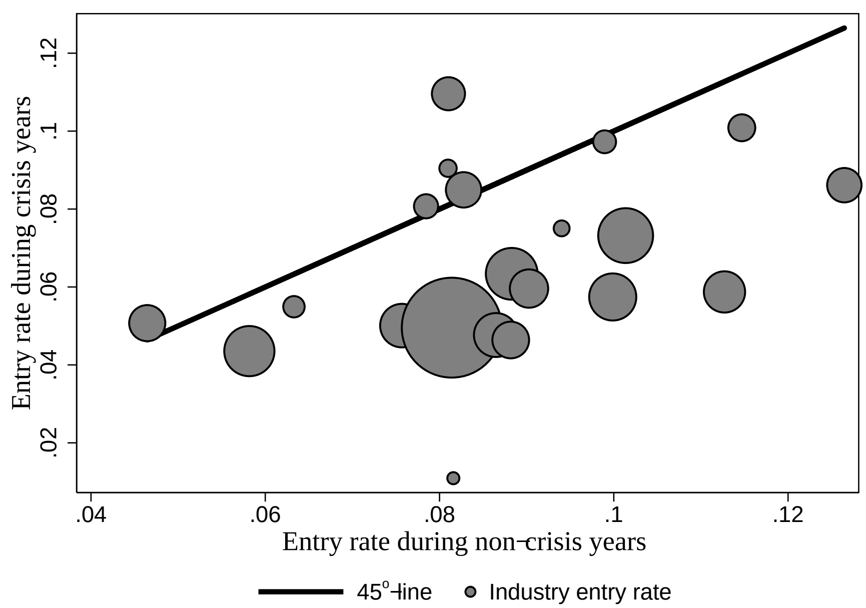


FIGURE III

Mass effect: Industry-level decrease in entry rates during the crisis.

Notes: The figure presents average entry rates for Chilean manufacturing industries over two set of years: no crisis (1996–1997, 2001–2005), and crisis (1998–2000), illustrating the adverse effect of the Chilean sudden stop on firm entry. The size of the markers represents the labor share of each 3-digit industry in 1995. The weighted average decrease of entry during years of crisis is 2.6 percentage points, or 29.3% of the pre-crisis weighted average.

For most industries, the average entry rate of 1998 through 2000 is significantly lower than in other years. Although it is clear that *fewer* firms are born during the crisis, we still have to analyze whether they are *better*. To capture the firm's quality we calculate firm level productivity applying the Wooldridge (2009) extension of Levinsohn and Petrin (2003)'s methodology at the three-digit industry level.[21] Denote the estimated log TFP of

is not in the panel. We also exclude entrants in the last year (2007) to avoid end of panel irregularities. Results are robust to including this last year.

[21]Because of the lack of price variables, our empirical productivity measure is revenue TFP. See Foster et al. (2016) for a useful discussion of revenue-based measures.

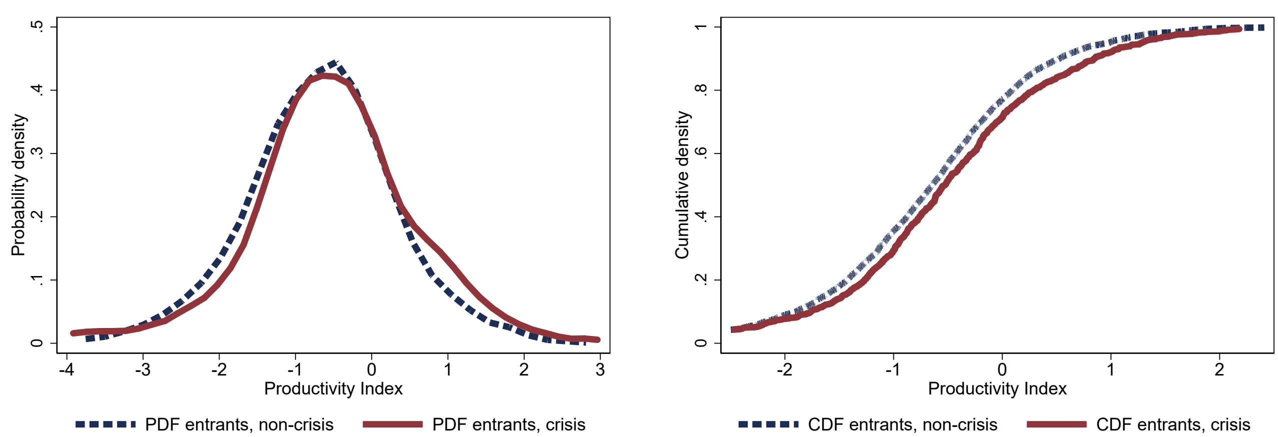
firm i in industry j at time t by $A_{i,j,t}$. Denote by $\theta_{i,j,t}$ the labor share of firm i in industry j at time t. Then, following the productivity literature, Foster et al. (2001), we can construct an index of industry productivity as:

$$A_{j,t} = \sum_i \theta_{i,j,t} A_{i,j,t}. \tag{33}$$

Furthermore, define the relative productivity of a firm with respect to its industry-weighted mean as:

$$\hat{A}_{i,j,t} = A_{i,j,t} - A_{j,t}. \tag{34}$$

Figure IV shows the PDF and CDF of $\hat{A}_{i,j,t}$ for entrant firms (age 0) pooling observations in crisis and non-crisis years. The composition effect can even be seen in the raw data. The distribution of relative productivity $\hat{A}_{i,j,t}$ shows that firms born during crisis are more productive than firms born during normal times. In fact, the mean of relative productivity is 16.6 percentage points larger for the crisis distribution.



(A) Probability density function

(B) Cumulative density function

Figure IV

Composition effect: Conditional productivity distributions.

Notes: The figure shows the PDF and CDF of relative TFP distribution for entrant firms (age 0), pooling observations in crisis and non-crisis years. The mean relative productivity is 16.6 percentage points larger for the crisis distribution.

In order to formalize the analysis, we define a *superstar* entrant as a firm with relative productivity $\hat{A}_{i,j,t} > \frac{\sigma_{j,t}}{2}$ on its first operation year, where the time-variant industry-level dispersion of productivity $\sigma_{j,t}$ is

$$\sigma_{j,t} = \sqrt{\sum_i \theta_{i,j,t} \left(\hat{A}_{i,j,t}\right)^2}. \tag{35}$$

We estimate the probability of being a superstar firm using the following logit specification:

$$Pr(\hat{A}_{i,j,t} > \frac{\sigma_{j,t}}{2}|\text{age} = 0) = \frac{e^{x_i'\beta}}{1 + e^{x_i'\beta}}; \quad x_i'\beta = \alpha + \alpha_j + \alpha_r + X'\omega + \gamma_{\text{cohort}} + u_{i,t}, \tag{36}$$

where α_j is a three-digit industry control, α_r is a geographical control, and X represents other potential controls.[22] In the baseline specification, the cohort coefficient indicates whether a firm was born during the sudden-stop window. Table I presents the results for five alternative specifications.

Table I
Probability of a Superstar Firm

$Pr(\hat{A}_{i,j,t} > \frac{\sigma_{j,t}}{2}\|\text{age} = 0)$	(1)	(2)	(3)	(4)	(5)
In Crisis	0.442***	0.497***	0.508***	0.551***	
	(0.095)	(0.118)	(0.117)	(0.154)	
After Crisis		0.087	0.099	0.087	
		(0.108)	(0.108)	(0.136)	
Avg entry$_{j,0}$					-2.602**
					(1.111)
HHI$_{r,j,0}$			-1.980**	-2.606**	-2.644**
			(0.864)	(1.018)	(1.040)
log K$_{i,t_0}$				0.505***	0.499***
				(0.038)	(0.030)
Industry and region FE	Yes	Yes	Yes	Yes	Yes
Observations	4140	4140	4140	3889	3889
Relative effect at means (%)	45	53	55	69	

Robust standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The first regression compares cohorts born during the crisis (1998 to 2000) against every other cohort. Firms born during the crisis are statistically more likely to enter as superstars in their industries. In fact, the estimated probability of being a superstar is 18.3% for firms

[22]The mean and standard deviation are calculated using every plant operating in a given year. In particular, we do not drop the firms born before 1995 from the sample to calculate these moments.

born during the crisis, while the probability for a firm born outside this window is 12.6%; thus, firms born in crisis are 45% more likely to be superstars in their industry.[23] The second specification shows that allowing cohorts born before and after the episode to differ does not change the results.

Regressions (3) and (4) aim at controlling for two complementary explanations. First, firms entering during crisis could face very different competitive pressures than firms that enter in normal times, implying different business strategies (Sedláček and Sterk, 2017; Moreira, 2015). The third specification includes the Herfindahl-Hirschman concentration index (HHI) at the industry-region level of the first year of life for each firm so that competitive pressures at birth are controlled for. Second, differences in collateral availability could explain differential firm performance. The fourth specification takes into account collateral by controlling for the capital stock of the firm on its first period of operation. The coefficient of interest retains its significance and magnitude in both cases, and while more collateral seems to increase the probability of observing a superstar firm, more concentration (less competition) decreases the probability of observing a superstar entrant. The fifth specification studies the effect of the three-digit industry entry rate at the moment of entry. This industry-level entry rate is a continuous variable common to every firm in the same industry born in the same year. According to our model, smaller cohorts should be, on average, better. Therefore, the negative coefficient supports the *fewer, but better* hypothesis. Given the evidence presented in Figure IV, it is not surprising that these results are robust to alternative definitions of superstar entrants.

The predictions of the model are stronger than those analyzed in the regressions involving superstar firms. The model predicts that firms born during crises are, on average, more productive, not only during their first year of operation, but during their entire life. In this context, we make use of the panel dimension of the data. The variable of interest (condition at birth) is time invariant, and by construction, it is correlated with the unobservable fixed effect. In fact, we would like to isolate from the fixed effect of a firm the component that is due to a particular birth condition. Therefore, the main challenge of this panel estimation is that the variable of interest is not only time-invariant, but also endogenous. On the one hand, coefficients on time-invariant variables can be consistently and efficiently estimated by random effects regression, but the estimation is not consistent when the variable is also endogenous. On the other hand, fixed-effects panel regression can consistently estimate every

[23]The industry and region fixed effects are evaluated for the most populated industry (311) and the most populated region (central).

coefficient associated with the time-variant variables, but it cannot identify the coefficients of the time-invariant variables. In particular, we would like to estimate the following equation:

$$P_{i,t} = \alpha + \beta_1 X^1_{i,t} + \beta_2 X^2_{i,t} + \gamma_1 Z^1_i + \gamma_2 Z^2_i + \mu_i + u_{i,t} \tag{37}$$

where $X^1_{i,t}$ represents exogenous time-varying variables (four macro controls and the age of the firm), $X^2_{i,t}$ refers to endogenous time-variant variables (post-entry firm decisions), Z^1_i correspond to exogenous time-invariant variables (region and 3-digit industry-fixed effects), and Z^2_i are endogenous time-invariant variables (initial capital, HHI at birth, being born during a crisis, or industry entry rate at birth). Variables with a superscript "2" are endogenous in the sense that they are likely to be correlated with the unobserved fixed effect μ_i. In this situation, the Hausman and Taylor (1981) procedure delivers consistent and efficient estimators for every coefficient in equation (37).[24]

Table II presents the results for four different specifications. The dependent variable is $\hat{A}_{i,j,t}$, the relative productivity of firm i at time t with respect to the productivity index of its industry j. In the first regression, we use a single dummy to determine whether the cohorts born in 1998 through 2000 perform better than every other cohort. In the second regression, we use two dummies in order to allow a differential effect for cohorts *pre*- and *post*- crisis. The third specification studies the effect of the three-digit industry entry rate at the moment of entry. This industry-level entry rate is a continuous variable common to every firm in the same industry born in the same year and is also time-invariant. The fourth specification replicates the second including post-entry firm decisions (electricity consumption to proxy for utilization, capital stock, and total labor).

We circle back to our main question: Are those *fewer* firms born in crisis *better*? The first specification shows that firms born during the sudden stop have, on average, a productivity index 64% higher than firms born in normal times. This coefficient is robust to allowing for post-crisis cohorts to differ from before-crisis cohorts (specification 2). The third specification is more general in the sense that it aims to directly unveil a mass-composition

[24]See Online Appendix 2.5 for a succinct explanation. Intuitively, this procedure aims to remove the endogenous component from the original regression in order to meet the main assumption of random effects. More details on this method can be found in Wooldridge (2010), Chapter 11. STATA software has built-in routines for both procedures; see Schaffer and Stillman (2011). After every estimation, we perform the Sargan-Hansen test to assess the validity of the instrumental variables procedure at the core of Hausman and Taylor (1981). The null hypothesis is that the instruments are valid, so the higher the p-value, the better. Nevertheless, the richer the time-invariant endogenous variables are, the fewer degrees of freedom are left for the test. This is particularly relevant for the third specification of Table II.

TABLE II
HAUSMAN AND TAYLOR PANEL REGRESSION

$\hat{A}_{i,j,t}$	(1)	(2)	(3)	(4)
During Crisis	0.637***	0.618**		0.586**
	(0.227)	(0.275)		(0.245)
After Crisis		0.089		0.060
		(0.143)		(0.144)
Avg entry$_{j,0}$			-5.923***	
			(1.552)	
log age$_{i,t}$	0.006	0.008	0.022	0.014
	(0.022)	(0.022)	(0.021)	(0.021)
log K$_{i,t_0}$	0.796***	0.726***	0.561***	0.726***
	(0.136)	(0.190)	(0.066)	(0.206)
HHI$_{r,j,0}$	27.21**	19.86	9.870	26.76
	(13.30)	(21.26)	(11.22)	(20.86)
Industry and region FE	Yes	Yes	Yes	Yes
Macro Controls	Yes	Yes	Yes	Yes
Post entry controls	No	No	No	Yes
Observations	17646	17646	17646	17484
Sargan-Hansen (p)	0.495	0.242	0.014	0.205

Standard errors in parentheses (bootstrapped (250), clustered by firm)
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

tradeoff. The coefficient suggests that firms born in smaller cohorts have a permanently positive effect in their productivity measure. In particular, every extra percentage point in entry decreases the productivity index of the firm by 6%. Initial competitive pressures are only significant in specification 1, but the sign is reverted with respect to Table I, suggesting that more productive firms are able to enter and survive in more concentrated markets. More collateral at entry is still associated with higher productivity throughout a firm's lifespan.[25]

[25]One caveat related to *post-entry* selection can be added to the preceding results. If firms born during crisis are more likely to die early, then those cohorts would seem more profitable after that initial selection. Moreover, the model predicts that firms born during the crisis episode should be more resilient. Online Appendix 2.6 estimates a proportional hazard model in order to evaluate this concern. The main empirical question in the appendix is whether firms born during the crisis window are more likely to exit. The answer is negative.

In summary, the Chilean sudden stop had strong macroeconomic consequences. At the firm level, the effect is relatively more complex. Cohorts born during the crisis and in its aftermath are 30% smaller; nevertheless, the average firm born during the crisis is 64% more productive than the average firm born in normal times. Hence, taking the average quality of the entrant cohort as a reference to evaluate the losses from forgone entry is extremely misleading, as unborn firms are substantially *worse* than the observed ones, validating the main mechanism of this paper. The next section proceeds to calibrate the quantitative model and quantify the long-run cost imposed by a sudden stop when the mass and composition tradeoff is accounted for.

5. Quantitative Analysis

In this section, we explore the quantitative behavior of the model economy. First, we calibrate the model to the Chilean plant- and macro-level data. Second, we test the calibrated model using non-targeted firm dynamics and business cycle moments. The calibrated model delivers firm dynamics and macroeconomic aggregates that closely mimic their data counterparts. Third, we use the model to quantify the permanent productivity effect of the Chilean sudden stop and highlight the role of firm heterogeneity and selection. Fourth, we use cross-industry data to confirm that firm heterogeneity shapes the mass and composition effects of financial crises.

5.1. Calibration

5.1.1. Externally Calibrated Parameters

The 22 parameters of the model are calibrated on a quarterly basis. A first group of 12 parameters is externally calibrated according to the literature and features of the Chilean data. Table III presents the values for every externally calibrated parameter.

The labor share ($1-\alpha = 0.32$), the intertemporal elasticity of substitution ($1/\gamma = 0.6$), and the Frisch elasticity of labor supply ($1/(\chi - 1) = 2.2$) are set in accordance with the small open economy business cycle literature. The curvature of the expansion cost function of incumbents ($\xi = 2$) is taken from Akcigit and Kerr (2018) and their discussion of the empirical literature on endogenous technical change. We set the persistence of the stationary TFP process (ρ_z) to 0.95, in line with the literature. The depreciation rate of capital (δ)

is set at 8% annually, consistent with the study by Bergoeing et al. (2002) of the Chilean economy. The parameter governing the debt adjustment cost ($\psi = 10^{-4}$) is set to a low value that assures stationary behavior. We use interest payments and production costs from the Chilean micro data together with the series for the country interest rate to calculate the working capital requirement (η). Our calculations imply that 60% of the wage bill has to be kept as working capital.[26] We set $\bar{b}$ to match the average quarterly debt-to-GDP ratio of Chile (44% annual). We use average real interest rates for commercial loans with a maturity of one to three months from the Chilean Central Bank to estimate the parameters of the interest rate process for the period from 1996:Q2 through 2011:Q2. The estimation delivers $\bar{R}$ (5.5% annually), ρ_R (78% quarterly), and σ_R (0.46%).

Table III
Externally Calibrated Parameters

Parameter	Symbol	Value	Source
Capital share	$1-\alpha$	0.32	Mendoza (1991)
Elasticity of substitution ($1/\gamma$)	γ	1/0.6	Mendoza (1991)
Frisch elasticity ($1/(\chi-1)$)	χ	1.455	Mendoza (1991)
Expansion cost curvature	ξ	2	Akcigit and Kerr (2018)
AC stationary TFP	ρ_z	0.95	Neumeyer and Perri (2005)
Depreciation rate	δ	1.94%	Bergoeing et al. (2002)
Debt Adjustment Cost	ψ	10^{-4}	Stationarity
Working capital	η	0.6	Data
Long-run debt to GDP ratio	$\bar{b}$	$4*(-0.44)$	Data
Long-run interest rate	$\bar{R}$	1.0135	Data
AC interest rate	ρ_r	0.78	Data
Stdev interest rate	σ_r	0.46%	Data

5.1.2. Internally Calibrated Parameters

A second group of nine parameters is calibrated to salient features of both macroeconomic and firm-level data. The first seven parameters in Table IV are calibrated to the balanced growth path of the model. Although every long-run moment is related to the first seven parameters, we can point to some strong relationships between targets and parameters that identify the model. The mass of varieties (Λ) is used to normalize the mass of firms in the economy to unity. The disutility of labor (Θ) is set to match a long-run labor supply of

[26] Online Appendix 2.4 shows how this number is calculated. It is substantially lower than the 100% used by Neumeyer and Perri (2005) and the 125% used by Uribe and Yue (2006). Online Appendix 3.2.1 explores other values for robustness purposes.

33%. The average cost of starting a firm (κ) is related to the long-run entry rate; we set that target to a level consistent with the average entry of the pre-crisis years in our sample.[27]

In order to understand the identification of the parameters governing heterogeneity and firm dynamics, notice that, without heterogeneity and in continuous time, the balanced growth path of this economy collapses to a version of Klette and Kortum (2004), where the analytic size distribution is logarithmic.[28] Recall that in a logarithmic distribution, one parameter governs all the moments of the distribution. Introducing heterogeneity allows the model to target more than one moment of the size distribution. Intuitively, the size distribution of the model with two types can be thought as the combination of two logarithmic distributions. Thus, there are three degrees of freedom: the two parameters governing the distributions and the weights of each distribution.[29] Given these degrees of freedom, we identify the parameter governing the scarcity of high-type business ideas in the economy (ν) with the standard deviation of the firm's size distribution.[30] Because most of the firms are small, ν governs the composition of low- and high-type firms at small sizes. The productivity improvement that characterizes high-type firms (σ^H) determines the annual growth rate of the economy. Because high-type firms expand faster and live longer, their step size is key for the long-run growth of the economy. The step size of low-type firms (σ^L) is related to the mean of the size distribution. In fact, given σ^H, we adjust σ^L to match the average number

Table IV
Internally Calibrated Parameters

Parameter	Symbol	Value	Main identification	Target	Model
Mass of Varieties	λ	6.96	Mass of firms	1.00	0.95
Labor disutility level	Θ	30.67%	Working time	33.00%	31.42%
Entry Cost	κ	5.08%	Entry rate	10.80%	10.63%
Step Size H	σ^H	7.20%	Annual GDP Growth	3.00%	2.87%
Step Size L	σ^L	7.29%	Mean of firm employment distribution	7.62	7.35
Scarcity	ν	54.62	Stdev of firm employment distribution	13.29	12.74
Expansion Cost scale	φ	26.99%	Share of employment of 10% largest firms	48.30%	49.72%
Stdev TFP	σ_z	1.22%	Quarterly output volatility (HP filtered)	3.00%	3.00%
Capital adjustment cost	ϕ	9.50	Quarterly investment volatility (HP filtered)	9.56%	9.56%

[27]To be consistent with the annual frequency of ENIA we measure entry annually in the model. The target is the weighted average (labor) of pre-crisis 3 digit entry rates.

[28]See Appendix 1.2 for details.

[29]The first two are related to the endogenous expansion rates of firms, and the third one is related to the endogenous share of high-type firms in the economy.

[30]The size distribution is measured in terms of workers per firm and normalized such that the smallest firm has size 1. The same normalization is applied to the model.

of workers per firm in the data. Because the scale parameter of the cost of expanding (φ) governs the shape of the right tail of the size distribution, we discipline it by targeting the labor share of the largest 10% of firms.

The last two calibrated parameters are set to match business cycle moments of the Chilean economy. The standard deviation of the aggregate productivity disturbance (σ_z) and the parameter governing the capital adjustment cost (ϕ) are set to match the volatility of the HP filtered series of manufacturing output and economy wide investment between 1996:Q1 and 2011:Q2, respectively.[31] The model is able to match the targets successfully. Table IV presents the performance of the model regarding the nine targets and the corresponding values for each parameter.[32] Finally, the patience parameter (β) is set so that there is no bond-holding cost paid along the balanced growth path.[33]

The scarcity of good ideas implies that, under random selection, only $\underline{\tilde{\mu}} = 1.8\%$ of ideas would generate a high-type firm. Nevertheless, because the financial intermediary sets its cutoff to accept only the top 2.9% of business ideas, financial selection implies an ex-post fraction of high types of $\tilde{\mu} = 50.2\%$ at entry. The value of Λ implies that the average firm has seven products. Among the unit mass of firms, 57.3% are high-type firms, and they dominate $\mu = 74.2\%$ of the products. Although the two step sizes seem to be very similar, they imply extremely different firm dynamics. In fact, low-type firms expand at a rate $\iota^L = 9.31\%$ achieving an average size of only 4.4 product lines, while high-type firms expand at a rate $\iota^H = 9.82\%$ achieving an average size of 9.5 product lines. Because every product line is lost with probability $\Delta = 10.1\%$, high-type firms, being larger on average, survive longer.[34] The next subsection evaluates the performance of the model using non-targeted moments.

5.2. *The Micro- and Macro-performance of the Model*

Before using the model to assess the productivity cost of a sudden stop, we evaluate the calibration by studying the ability of the model to match firm-level and business cycle moments that were not used in the calibration procedure. We start by evaluating the long-run calibration of the model using micro data, a dimension about which a standard small open economy model would be silent. Figure V evaluates the performance of the model

[31] There is no quarterly data on manufacturing investment for Chile.

[32] To calibrate, we minimize the sum of the absolute values of the percentage deviations of the model-implied moments from targets.

[33] From the bond-holding first-order condition, we obtain $\beta = \frac{(1+a)^\gamma}{R}$.

[34] Subsection 5.4 shows that ignoring heterogeneity has large quantitative effects when studying a crisis.

with respect to the size distribution.[35] The model successfully tracks the complete size distribution of firms in 1995.

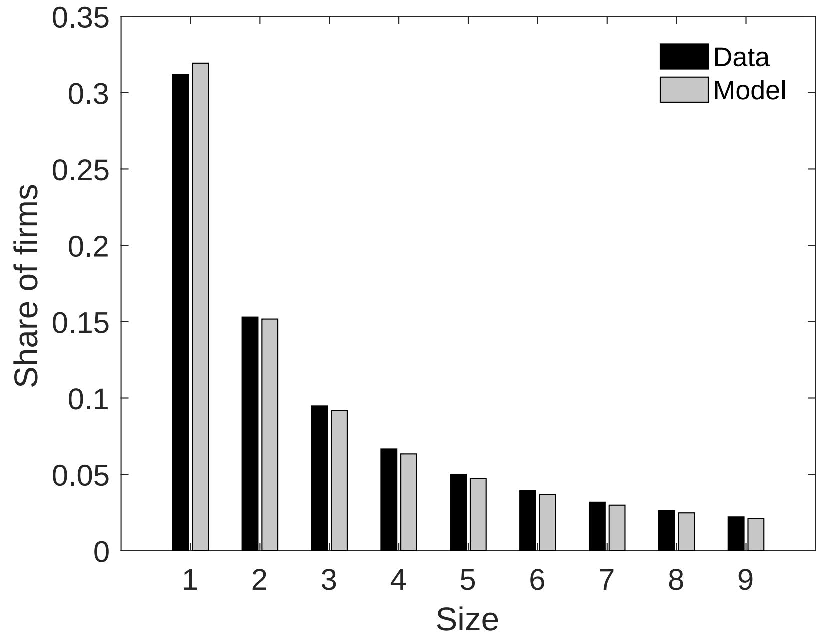


FIGURE V
Firm-level validation: size distribution

Having established the ability of the model to generate a stationary size distribution aligned with the micro data, we evaluate the business cycle dynamics of the model. First, Table V shows the standard deviation and autocorrelation of the hp-filtered series of log-output, log-labor, log-consumption, and log-investment.[36] Only the standard deviation of output and investment (highlighted in bold) are targeted in the calibration. The business cycle moments are consistent with the behavior of the Chilean economy. Second, because interest rate fluctuations play a fundamental role in this paper, we also compare the correlation of the main macro variables with the lagged interest rate. The final two columns of Table V shows that the model is consistent with the countercyclical interest rate of the Chilean economy.

[35]Firm level employment is relative to the employment of the smallest firm in model and data.

[36]The hp parameter is set to 1600 in accordance with the quarterly frequency. Due to the lack of quarterly data for manufacturing capital and labor we use the total capital stock and total hours worked for the Chilean economy along with quarterly manufacturing output. Because we compare deviations from an HP-trend this exercise is valid as long as the cyclical behavior are similar. Every result in the paper is robust to using total quarterly output instead of manufacturing output.

Table V
Autocorrelation and Standard Deviations

	Autocorr		Standard dev.		Corr(x,R(-1))	
	Data	Model	Data	Model	Data	Model
Output	0.650	0.702	**0.030**	**0.030**	-0.001	-0.100
Labor	0.590	0.709	0.017	0.021	-0.301	-0.290
Consumption	0.750	0.720	0.027	0.024	-0.318	-0.371
Investment	0.620	0.618	**0.096**	**0.096**	-0.452	-0.534
TFP	0.548	0.697	0.016	0.016	0.134	-0.017
Entry	-0.092	-0.068	0.018	0.095	-0.551	-0.662
Exit	-0.334	-0.049	0.017	0.015	-0.242	-0.053

Notes: All series but the entry and exit rates refer to hp-filtered log levels at quarterly frequency. *corr(x,R(-1))* denotes the correlation between a variable and one-period lagged interest rate. Firm entry and exit rates are at annual frequency and hp-filtered.[37] Figures in bold refer to targeted moments.

More importantly, we evaluate the ability of the model to replicate the entry and composition dynamics observed during the crisis. Because the model has only two exogenous shocks, only two series can be perfectly targeted. We assume that the model is on a balanced growth path in 1996:Q1, and we use the output and interest rate deviations in the data to filter through the model productivity and interest rate innovations.[38] To inform the model, we use the demeaned series of log differences of output and the demeaned series of the log of $R(s^t)$.[39] Figure VIa shows that the annual entry rate behavior in the model is aligned with the U-shape behavior observed in the data during the crisis.[40] The model successfully replicates the drop in entry during the Asian crisis in 1997 and the Russian default in 1998 and the pickup afterwards, albeit with a somewhat larger magnitude.

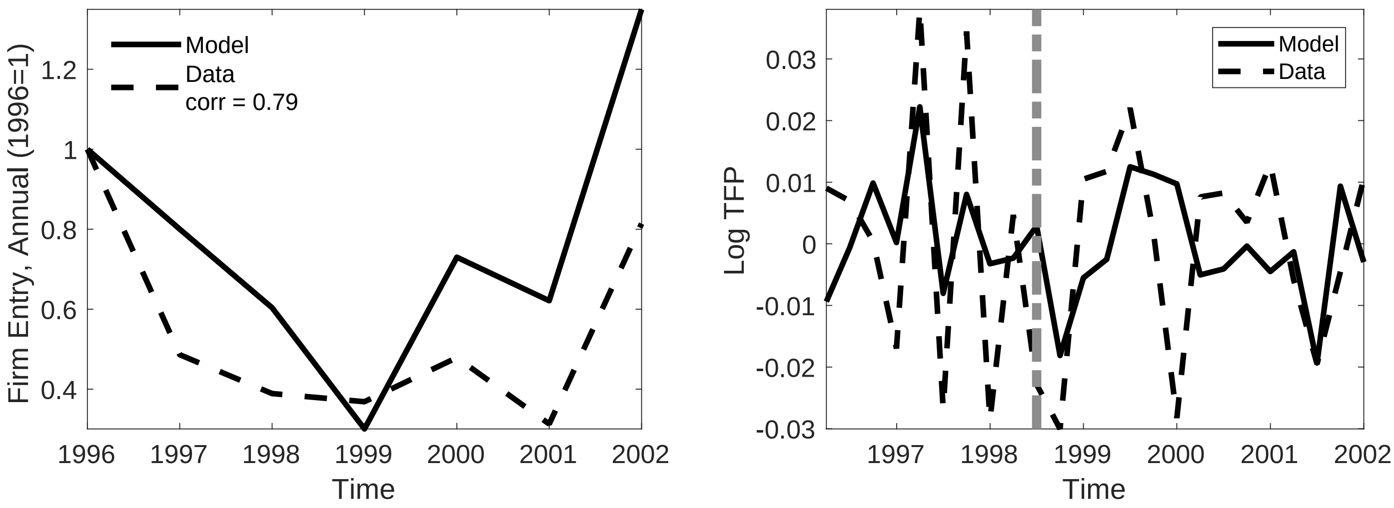
We evaluate the macroeconomic performance of the model further by zeroing in on the

[37]We chose to present results based on the series at the annual frequency to facilitate the comparison with the data, as we do not have quarterly data for these variables. However, this leaves us with limited number of observations, which is not ideal for hp-filtering. Unreported results show that the model mimics the data closely also when we consider variables at levels, with the autocorrelation coefficient of the entry rate turning positive both in the data and the model. The model-generated quarterly series has a similar coefficient of correlation with the lagged interest rate.

[38]The model is solved by second-order perturbations using Dynare. The model has no kinks in value or policy functions. As discussed in Aruoba et al. (2006), higher order perturbation methods are appropriate for smooth systems with strong nonlinearity subject to large shocks. When filtering shocks, we use a first-order solution and a Kalman filter.

[39]Online Appendix 3.1 shows the data and filtered shocks used for this exercise.

[40]The data counterpart is a weighted average of the three digit industry entry rates using labor shares in 1995 as time-invariant weights. Year 1996 is normalized to unity in model and data. Appendix 1.2 also shows that the model replicates the flatter exit rate dynamics.



(A) Non-Targeted Entry Rate

(B) Solow Residual

Figure VI
Firm Creation and Solow Residual

Notes: The left panel compares annual firm entry rate in the data and the model over the crisis. The right panel shows the HP-filtered Solow residuals for model and data.

Solow residual. In the model, we measure the Solow residual $S(s^t)$ as

$$\ln S(s^t) = \ln Y(s^t) - \alpha \ln L(s^t) - (1-\alpha) \ln K(s^t). \tag{38}$$

Figure VIb compares the model-implied Solow residual to its empirical counterpart during the crisis. We compute the empirical Solow residual for the whole economy using aggregate output, capital, and labor because we lack the measure of inputs at the industry level and quarterly frequency. Thus, the filtered manufacturing output (matched by construction) is not part of this validation exercise. As shown by the graphs, the calibrated model is able to track most of the fluctuations of the aggregate Solow residual during the crisis, only with slightly lower volatility relative to the data. Table V show that the time series properties of firm entry, exit, and Solow residuals in the model are also broadly consistent with the data.

In order to assess the ability of the model to generate the increase in superstar firms during the crisis, we define the firm-level revenue productivity of a type d firm $\hat{q}^d$ as

$$\hat{q}^d_f \equiv \ln\left(\sum_{j \text{ owned by } f} p^d(s^t)X^d(s^t)\right) - \ln\left(nl^d(s^t)\right) = \ln\left(w\left(1+\sigma^d\right)\right). \tag{39}$$

Table VI compares the baseline empirical regression with the same specification using simulated data from the model. Model and data generate a positive and significant increase in the probability of observing a superstar firm during the crisis. Because the model has only two step sizes, there are only two possible revenue productivity levels.[41] The calibration of the model implies that high-type firms are superstars according to the definition used in the empirical section. As a consequence, the unconditional probability of observing a superstar firm in the model is larger than in the data, triggering the difference in the absolute increases captured by the regression coefficient. Nevertheless, when the regressions are evaluated at their mean and we calculate the percentage increase in the probability of observing a superstar firm, model and data imply that during crisis, it is 45% more likely to see a superstar than during normal times.

TABLE VI
PROBABILITY OF A SUPERSTAR FIRM

$Pr(\hat{A}_{i,j,t} > \frac{\sigma_{j,t}}{2} \vert \text{age} = 0)$	Data	Model
In Crisis	0.442^{***}	1.880^{***}
	(0.095)	(0.072)
Relative effect at means	45%	44%

Robust standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In sum, the calibrated model is able to mimic the patterns of firm dynamics, the business cycle behavior, and the mass and composition of entrant cohorts—stylized facts documented in Section 4. Establishing this, we proceed to use the model for quantifying the permanent productivity cost associated with the Chilean crisis.

5.3. *The Permanent Productivity Loss of a Sudden Stop*

Having validated the calibrated model, we use the model economy to study the dynamics of the endogenous productivity $A(s^t)$ and the permanent output loss due to the slow down in firm dynamics during the sudden stop. The logarithm of total factor productivity (TFP) is given by

$$\ln TFP(s^t) = z_t + \alpha \ln A(s^t), \tag{40}$$

[41] The empirically unobserved *quantity* productivity is $\bar{q} = \ln\left(\frac{\sum_{j \text{ owned by } f} q}{n}\right)$ and it has a rich distribution.

where the second term represents the endogenous productivity component, which grows at rate $a(s^t)$ as defined in equation (32). Recall that output and consumption are normalized by $A(s^t)$; therefore, a 1% permanent loss in $A(s^t)$ generates a 1% permanent loss in consumption but only a α% loss in TFP.[42] To better understand the dynamics of these permanent losses, we first examine its drivers.

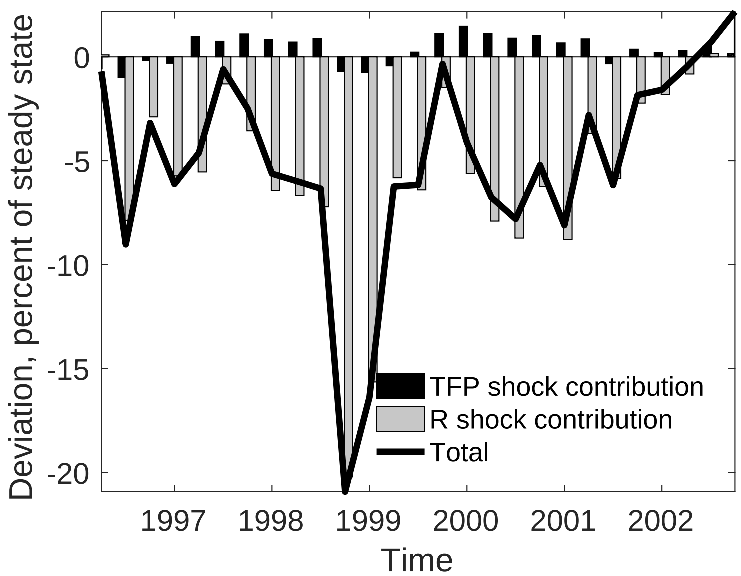


FIGURE VII
Shock Decomposition of $a(s^t)$

Notes: The figure shows the contribution of aggregate productivity and interest rate shocks to the dynamics of the growth rate of endogenous productivity.

Permanent losses in $A(s^t)$ result from the deviations of its growth rate $a(s^t)$ from its long-run trend, which accumulate over time to permanent-level changes. Therefore, to understand the drivers of the loss in $A(s^t)$, we focus on the drivers of fluctuations in $a(s^t)$. Accordingly, Figure VII shows the importance of innovations to exogenous productivity $z(s^t)$ and interest rate in explaining the deviations of $a(s^t)$ from its long-run trend. In line with the variance decomposition and the impulse response functions analysis in Online Appendix 3.1, the majority of the deviations in endogenous productivity growth are accounted for by the interest rate shock. Therefore, the calibrated model supports the view that financial shocks are special in terms of productivity dynamics. The dominant contribution of interest

[42]See Online Appendix 1.7 for a theoretical explanation of the difference between endogenous and exogenous growth and how it relates to the permanent productivity loss.

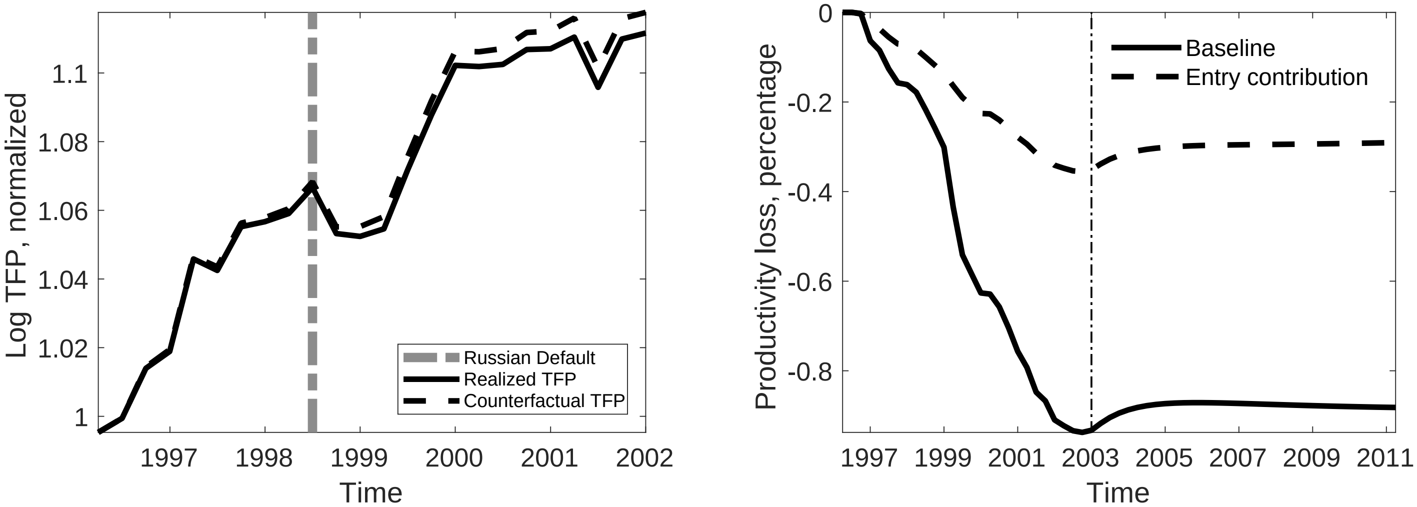
rate shocks to productivity fluctuations is driven by its differential impact on the stochastic discount factor of the household that determines the value of owning a variety. In fact, future payoffs are strongly discounted after an increase in interest rates but not after a negative productivity shock.[43]

We now turn to the quantitative importance of endogenous growth dynamics for aggregate TFP. Figure VIIIa compares the baseline TFP path generated by the model (solid line) to a counterfactual path (dashed line) that assumes constant BGP growth in $A(s^t)$, abstracting from the fluctuations in $a(s^t)$ depicted in Figure VII. Both paths are in logs and normalized to 1 in 1996:Q1. The difference between the two paths captures the accumulated permanent TFP loss due to the endogenous productivity dynamics triggered by the otherwise stationary shocks, as the counterfactual path assumes a constant growth rate in $A(s^t)$. The results indicate that, by 2002, the accumulated permanent loss is 0.6% of TFP or, equivalently, 0.9% of output and consumption. These magnitudes are also in line with the findings of the existing empirical literature on the output and productivity losses associated with economic crises.[44] As such, the model provides a micro-founded theory that allows stationary shocks to have permanent effects due to endogenous trend dynamics.

Figure VIIIb looks further into the evolution of endogenous productivity $A(s^t)$, showing the permanence of the productivity loss and the role played by firm entry and the expansion decision of incumbents. First, the solid line tracks the accumulated percentage deviation of $A(s^t)$ from its counterfactual BGP path. To generate this path, we feed to the model the filtered shocks up to the last quarter of 2002, and then leave the economy converge freely to a new path absent any further interest rate and productivity shock. The graph shows that the endogenous productivity loss does not vanish after 2002. In the long run, the economy converges to a new path where endogenous productivity is permanently 0.9% below what it would have been in the absence of shocks. Therefore, absent new shocks beyond 2002,

[43] This is not related to the working capital constraint. Online Appendix 3.2.1 shows that eliminating the working capital channel does not affect the quantification of the permanent productivity loss of the crisis. This result is not surprising, given that the pass-through of interest shocks due to the working capital constraint is isomorphic to stationary productivity fluctuations. The magnitude of the working capital constraint only affects the short-run behavior of the economy.

[44] Cerra and Saxena (2008) document an output hysteresis for Latin America from a currency crisis between 4% to 6%, and between 1% to 6% for a banking crisis. Cook and Devereux (2006) analysis of the Asian crisis is consistent with this range pointing to a persistent output loss of 6%. Meza and Quintin (2007) report a similar number. After adjusting for capital utilization and labor hoarding, they conclude that one-fifth of the 6% is purely due to productivity. Therefore, the comparison point seems to be around a 6% loss in output where real productivity fluctuations should account for around 1% to 2%. Given that the crisis in Chile was milder than in its neighbors and the Asian countries, we consider that the model finding is the right empirical range.



(A) Measured TFP and Loss due to Exogenous Shocks

(B) Sources of Endogenous Productivity Loss

FIGURE VIII
Loss in Endogenous Productivity and Measured TFP

Notes: The left panel shows the percentage deviation of the growth rate of endogenous productivity component from its balanced-growth-path value, along with the respective contributions from each exogenous shock. The right panel shows the percentage loss in the endogenous productivity component with respect to its path along the balance growth path (solid line), and the loss that would have occurred had only the entry component responded to shocks.

consumption, output, and investment would be permanently 0.9% lower.

In order to study the role of entrants and incumbent firms in this process, we use the analytic expressions for the growth rate and the evolution of the composition of incumbents to separate two sources of the productivity loss: fluctuations on the entry margin and changes in the expansion decisions of incumbents. In particular, the dashed line in Figure VIIIb fixes the expansion rate of incumbents $\iota^d(s^t)$ in equations (30) and (32) but allows the composition of incumbents $\mu(s^t)$ to evolve according to the evolution of the composition of entrants $\tilde{\mu}(s^t)$. Therefore, the dashed line measures the direct effect of the entry margin, whereas the difference between the two lines captures the loss due to fluctuations in incumbents' dynamic decisions. Overall, one-third of the total productivity cost is generated by entry dynamics, while the rest is accounted for by fluctuations in incumbents' decisions.

In summary, four years after the onset of the sudden stop, the Chilean economy has lost 0.6% in terms of TFP and 0.9% in terms of output due to the permanent decline in endogenous productivity. This permanent loss is practically independent from the short-run stationary TFP shocks and is mostly due to the interest rate fluctuations during the crisis. We showed that, while fluctuations on the entry margin explains one-third of the TFP loss, the largest portion of this loss stems from disruptions in incumbent firms' decisions. In Online Appendix 3.2, we provide a sensitivity analysis of this productivity loss over the business cycle—along with other key variables—to various model parameters. In particular, we observe that the magnitudes are most sensitive to parameters that define innovation production (κ, ξ, φ).[45] We also analyze the robustness of our findings on permanent productivity loss driven by sudden-stop episodes to various modeling choices.[46] We refer the interested reader to Online Appendix 3.4 for the details of these robustness exercises.

To better understand the importance of heterogeneity and firm dynamics when evaluating the productivity cost of a crisis, in the next subsection we compare the baseline model with alternative economies that lack those dimensions.

5.4. The Role of Heterogeneity and Firm Dynamics

In this subsection, we highlight the role of heterogeneity and firm dynamics when quantifying the productivity cost of a financial crisis. In particular, we compare the baseline model to two alternative economies that feature endogenous growth: An economy with no incumbent dynamics and no heterogeneity (NDNH) and an economy with incumbent dynamics and no heterogeneity (NH). NH differs from the baseline version in that it has a single step size, removing heterogeneity and selection from the economy. NDNH goes one step further by removing the expansion decision of incumbent firms. Therefore, in this version, incumbent firms do not expand or shrink, holding one variety until they are replaced. We also include a fourth economy with exogenous growth (Exo).

The idea of the exercise directly relates to the question of the paper—that is, what are the long-run productivity effects triggered by a financial crises and heir determinants? We analyze the same data through the lens of each model and explore their long-run implications in order to understand the importance of the missing channels in each version for the long-run productivity effects. In particular, every model has the same interest rate process, the same output and investment volatility, and the same long-run growth rate. Models featuring

[45] Among these parameters, we only take the curvature of the incumbent R&D function from the literature. While this quadratic functional form that we adopt is supported by empirical evidence from the micro data (see, e.g., the discussions by Acemoglu et al., 2018 and Akcigit and Kerr, 2018), our results emphasize the importance of getting direct estimates of parameters that determine the probability of receiving successful innovations.

[46] In particular, we focus on three variations: i) capital investment for R&D instead of labor, ii) convex entry cost, and iii) and truncation of the model economy by firm size, addressing selection into the data based on size.

endogenous growth also have the same entry rate in the long-run. Online Appendix 3.3 explains the details of these alternative models and their calibrated parameters.

To highlight the importance of heterogeneity and firm dynamics, Table VII shows the permanent productivity loss and the consumption equivalent welfare cost across models when the economy faces a one-time mean-reverting interest rate shocks of 100 basis points.

Table VII

Long-run output and welfare cost of a 100 bps R shock

	Base	Exo	NH	NHND
Long-run Cost (LRC)	-0.24%	0%	-0.43%	-0.86%
LRC rel. to Baseline	100%	0%	174%	354%
Welfare (CEQ)	-0.15%	-0.10%	-0.19%	-0.26%
CEQ rel. to Baseline	100%	69%	125%	174%

Notes: LRC and CEQ stand for long-run cost and consumption equivalent welfare cost, respectively. A negative x% for LRC means that the endogenous productivity is x% lower than the un-shocked path in the corresponding model 1200 periods after the shock hits. A negative x% for CEQ implies that the representative household in the shocked economy would have the same welfare if she consumed x% less in the un-shocked economy. Model NH lacks heterogeneity in firm types, whereas model NHND lacks both type heterogeneity and incumbent expansion. Model Exo has a fixed growth rate that is equal to the balance growth path level in the baseline economy.

Recall that, in an economy with endogenous growth, every growing variable is subject to the same permanent level loss triggered by the adverse productivity dynamics. When productivity is exogenous, there is no permanent loss by construction. Ignoring heterogeneity (NH) implies a 50% larger long-run loss in output, and when heterogeneity and firm dynamics are omitted (NHND), the loss is more than three times the figure in the baseline. The main reason behind this upward bias is that, in the NH economy, funded business ideas are just as good as the discarded ones. This homogeneity implies that the marginal contribution of business ideas is flat with respect to the entry rate. Therefore, entry declines in NH more than in the baseline. To understand the large effect in NHND, recall that, by construction, in a model with no firm dynamics, entrants account for all the productivity growth in the economy. Because the crisis moves the entry margin violently, the effect on endogenous productivity is also outsized.[47]

[47] These extreme reactions are behind the large cost of business cycles found by Barlevy (2004) when introducing endogenous growth in an RBC model with no incumbent dynamics and no heterogeneity. This finding could also suggest that emerging economies with less heterogeneity and less post-entry firm-level growth can exhibit more endogenous fluctuations in trends, providing a micro foundation for the exogenous trend dynamics in Aguiar and Gopinath (2007).

To understand the economic importance of heterogeneity and firm dynamics, we calculate the consumption equivalent welfare loss associated with the interest rate shock. First, the agent in the baseline economy is willing to forgo 27% more consumption than the agent in the exogenous productivity model (Exo) in order to avoid the shock. Therefore, the welfare implication of the endogenous permanent productivity loss is relevant when compared to the standard short-lived effects of a crisis. As such, by ignoring the medium and long-run effects of a crisis, the standard small open economy model significantly underestimates the welfare cost of a crisis. Second, abstracting from incumbents' dynamics and firm heterogeneity increases the consumption equivalent welfare cost by 60% when compared to the baseline economy. Thus, introducing endogenous growth without modeling heterogeneity and firm dynamics overestimates the welfare costs of a crisis.

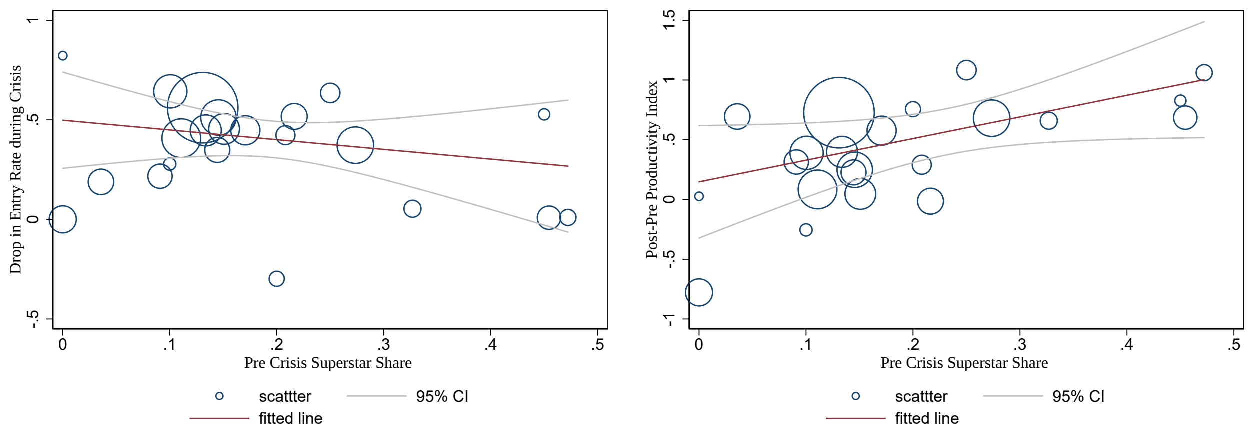
5.5. Empirical Evidence on Industry Heterogeneity and Selection

Contrasting the baseline model with a model that lacks heterogeneity, we conclude that economies with more heterogeneity decrease entry less during crisis due to the composition effect. The same logic implies that industries with more selection should see smaller declines in entry and less detrimental effects on average productivity during the sudden stop. We revisit the Chilean firm-level data to test these predictions.[48]

The mass and composition tradeoff suggests that industries with higher share of superstar firms in their entry cohort feature more selection. To evaluate this empirically, we calculate the fraction of superstars—firms that satisfy $\hat{A}_{i,j,t} > \frac{\sigma_{j,t}}{2}$—in a given cohort and average this measure over the pre-crisis years (1996–1997). This fraction of superstar entrants serves as our proxy for the strength of selection in each industry before the unexpected shock (Russian crisis) hits.

First, we test if industries with more selection actually observed a smaller decrease in entry rate. We define the drop in entry during crisis as the difference between the average industry entry rate in the pre-crisis years (1996–1997) and the average crisis year industry entry rate (1998–2000). In line with our conjecture, Figure IXa shows that the decrease in

[48] While the empirical analysis here makes use of multiple sub-sectors in the manufacturing industry, the quantitative model does not consider different sectors. Consequently, the mapping between the model and the empirical analysis is not exact because of the general equilibrium effects across industries existent in the data, which operate through wages and prices. However, the regression analysis presented in this section controls for fixed industry effects, correcting for time invariant level differences in these general equilibrium effects.



(A) Entry drop and selection

(B) Change in productivity and selection

FIGURE IX
Selection across industries

Notes: The regressions is a weighted OLS using industry employment in 1995 as weights.

entry during the crisis is indeed larger in industries that are characterized by less selection.

Second, we study the evolution of average industry productivity in relation to industry-level selection, for which we calculate pre-crisis (1996–1997) and post-crisis (2001–2006) productivity index $A_{j,t}$ as defined in equation (33). Figure IXb shows that the difference in $A_{j,t}$ before and after the crisis is larger in industries with more selection (a positive difference implies higher average industry productivity after the crisis relative to the pre-crisis average). Therefore, industries with larger pre-crisis superstar shares exhibit larger increases in average firm productivity during the crisis, consistent with more selection occurring during the sudden stop.

Finally, we make further use of pre-crisis industry variables to analyze the relationship of selection and heterogeneity with productivity at the industry level. Specifically, we estimate again the baseline superstar specification in equation (36), allowing for interactions with pre-crisis industry characteristics. Because years 1996 and 1997 are used to calculate pre-crisis industry characteristics, we avoid endogeneity concerns by estimating the regression between 1998 and 2006. Table VIII presents the results of this exercise.

The first specification confirms that crisis cohorts are more likely to give rise to superstar firms than post-crisis cohorts. This result establishes that dropping the two pre-crisis cohorts does not affect our results. The other three specifications include interactions of pre-crisis industry characteristics related to heterogeneity and selection with the crisis dummy.

Table VIII
Superstar and selection

$p\left(\hat{A}_{i,j,t} > \frac{\sigma_{j,t}}{2}\right)$	(1)	(2)	(3)	(4)
Crisis Dummy	0.398***	-0.350*	1.780***	1.124**
	(0.104)	(0.204)	(0.474)	(0.505)
Crisis Dummy × Superstar$_{j,pre}$		3.987***		4.068***
		(0.916)		(0.909)
Crisis Dummy × Entry$_{j,pre}$			-12.38***	-13.33***
			(4.160)	(4.286)
Industry and region FE	Yes	Yes	Yes	Yes
Observations	2981	2981	2981	2981

Robust standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The three-digit industry-fixed effect absorbs the level effect of these characteristics, leaving only the interaction. In line with the model intuition, industries characterized by more selection (higher pre-crisis superstar shares) and industries that rely less on the mass effect (lower pre-crisis entry rates) show larger increases in the share of superstar firms during the crisis. These effects are economically relevant; under the second specification, an industry in the bottom 25% of Superstar$_{j,pre}$ has a 13% probability of seeing a superstar arising in normal times, and during crisis this probability increases only to 14%. When we consider an industry in the top 25%, the probability of observing a superstar firm is 16% in normal times, and it increases to 28% during crisis. Similarly, under the third specification, an industry in the top 25% of Entry$_{j,pre}$ sees its probability of a superstar firm arising increase by 4 percentage points during the crisis while an industry in the bottom 25% of that index sees an increase of 11 percentage points. Interestingly, the fourth specification shows that mass and composition industry characteristics have independent effects. In sum, the cross-industry analysis support the view that heterogeneity and selection shapes the dynamics of crisis. In particular, industries with less heterogeneity and selection see larger decreases in firm entry and average productivity during financial crises.

6. Concluding Remarks

In this paper, we revisit the economic consequences of sudden stops by considering the effect of a crisis on productivity growth. With that aim, we present an open economy endogenous growth model subject to interest rate and productivity shocks. The engine of growth in this economy is the creative destruction induced by new entrants and by the expansion of incumbents. Because potential entrants are heterogeneous and promising entrants are scarce, financial selection introduces a tradeoff between the mass (quantity) and the composition (quality) of the entrants. In particular, a crisis triggered by an interest rate shock tightens lending, giving rise to a smaller cohort of entrants that, on average, generate higher productivity gains. We use the Chilean sudden stop to test the main mechanism of the model. Our empirical analysis confirms that, although fewer firms are born during the crisis, they are better in that they contribute more to aggregate productivity. Because entrants become incumbents and make expansion decisions, this composition effect has persistent consequences in the economy. The calibrated model successfully reproduces non-targeted features of the firm-level dynamics and the business cycle behavior of the Chilean economy. The model reveals that firm dynamics and heterogeneity are critical when evaluating the cost of a financial crisis. In terms of welfare, an interest rate shock triggers a 74% larger consumption equivalent welfare loss in a model with no heterogeneity and no firm dynamics. Because governments often use forgone entry as a foundation for policy interventions, a correct assessment of the cost of foregone entry is critical. This model provides a tractable framework that future studies can use to evaluate those policies.

The scope of this model is far beyond sudden-stop episodes or the particular Chilean experience. We develop a framework that allows researchers to introduce firm heterogeneity, firm dynamics, and endogenous growth in any dynamic general equilibrium model with aggregate risk with only one extra state variable and no approximation in distributions. This class of models provides a natural bridge to reconcile firm-level micro data and macro dynamics, as firm-level data can be directly used to inform the parameters of the model. Therefore, this class of models can provide not only a microfoundation for trend shocks (Aguiar and Gopinath, 2007; García-Cicco et al., 2010) and disaster risk (Colacito and Croce, 2011; Gourio, 2012), but also a path to discipline these mechanisms with firm-level data.

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Appendices

1. Model

1.1. Model Diagram and Timing of Events

The diagram in Figure X illustrates the linkages in the model, which are summarized in the introduction to Section 3.

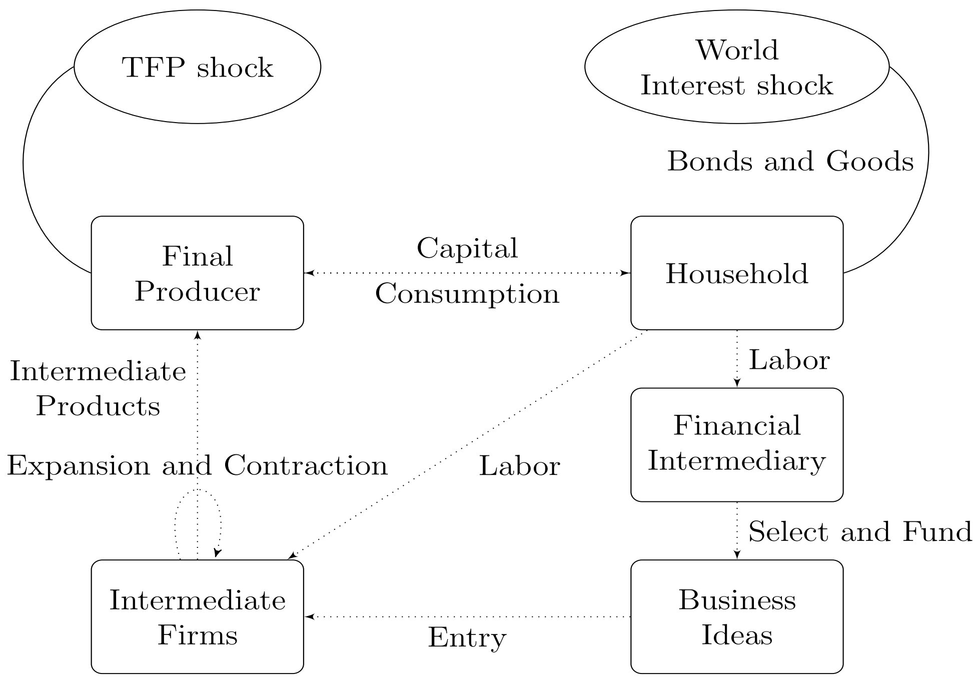


Figure X
Model Economy

Notes: The figure presents the building blocks and the flow of resources in the benchmark economy.

Figure XI summarizes the flow of events in a given period of time, dividing it into five consecutive intervals. First, the entry and the expansion chosen last period take place simultaneously. New entrants become single-product incumbents, and some of the former incumbents exit while others expand or contract. Second, the shocks are revealed, the final good producer demands capital and intermediate products, and the intermediate producers and the financial intermediary prepare to hire labor with intra-period borrowing in order to meet working capital requirements. Third, incumbents hire labor for production and expansion, and the financial intermediary hires labor to pay the entry cost of the selected business ideas. Fourth, production takes place and intra-period loans are repaid. Fifth, the representative household consumes, invests, repays last-period debt, and optimally chooses her bond holding for next period.

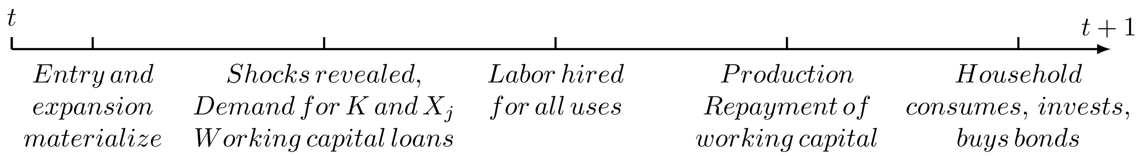
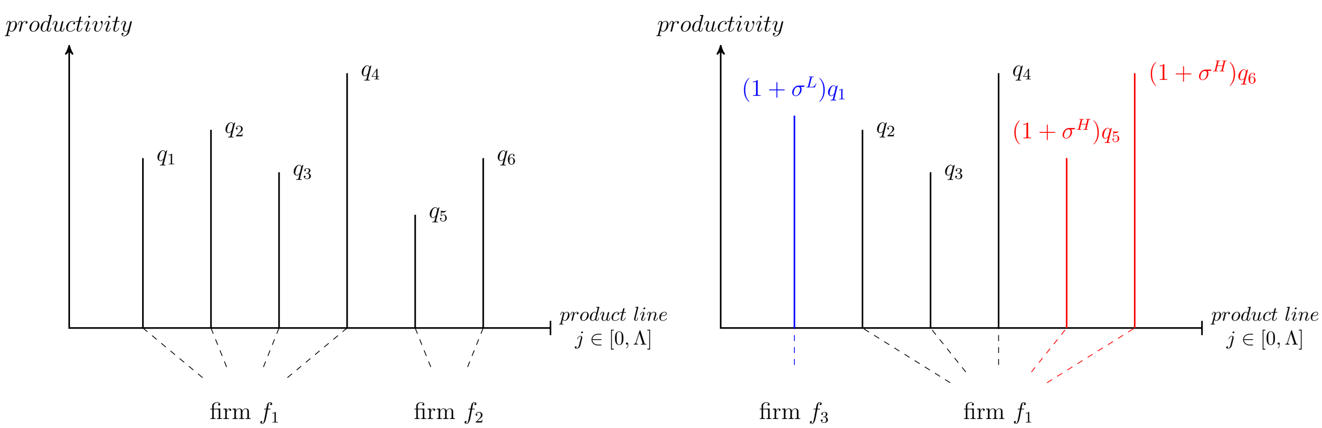


FIGURE XI
Timing Convention

1.2. *Firm Dynamics and Evolution of Firm Size*

Figure XIIa shows six product lines with different productivity levels and two firms. Firm f_1 owns four product lines, while firm f_2 owns only two. Initially, firm f_2 produces product 6 at the same productivity level at which f_1 produces product 1. Figure XIIb portrays the same products in the following period. Three elements are important to understand the dynamics of the model. First, firm f_1 has successfully expanded over the two products formerly owned by firm f_2, forcing f_2 to exit the market. Second, firm f_3 is a successful new



(A) Intermediate Product Firms (B) Innovation and Evolution of Firms

FIGURE XII
Evolution of firm size and their productivity levels

Notes: The left panel describes portfolio of product lines owned by a sample of intermediate producers and the associated productivity levels. The right panel describes how heterogeneous innovations shape the evolution of productivity levels as well as the ownership of these product lines by different firms.

entrant that has landed on product 1. Therefore, firm f_1 has lost her dominance over the first product. Nevertheless, by acquiring two new products, firm f_1 has expanded from five to six products on net. Third, f_1 is a high-type firm, whereas f_3 is of low type. This implies that products 1 and 6 no longer have the same productivity level, as they were subject to improvements of different scale.

The distribution of firms is determined by the mass of type d firms with n product lines at every aggregate state. We denote this mass by $\Omega_n^d(s^t)$, and the $\Omega_n^d(s^t)$ firms in this category control $\Omega_n^d(s^t) \cdot n$ product lines. Because time is discrete in this model, the changes in the number of product lines of each firm are described by a binomial process. The firms in the $\Omega_n^d(s^t)$ category might end up with any number of product lines in $[0, 2 \cdot n]$, depending on the interaction between their innovation effort and the replacement rate of the economy. For example, a firm with five product lines that successfully generates spinoffs in four of them but also loses two of its former products will end up with seven product lines. Therefore, we can use the law of large numbers to write the law of motion of each size class as below:

$$
\begin{aligned}
\Omega_1^H(s^t) = {} & M(s^{t-1})\tilde{\mu}(s^{t-1}) \\
& + \Omega_1^H(s^{t-1}) \sum_{k=0}^{1} \mathbb{P}\left(k, 1, \iota^H(s^{t-1})\right) \cdot \mathbb{P}\left(k, 1, \Delta(s^{t-1})\right) \\
& + \sum_{n=2}^{\infty} \Omega_n^H(s^{t-1}) \cdot \sum_{k=0}^{1} \mathbb{P}\left(k, n, \iota^H(s^{t-1})\right) \cdot \mathbb{P}\left(k+n-1, n, \Delta(s^{t-1})\right)
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
\Omega_1^L(s^t) = {} & M(s^{t-1})\left(1 - \tilde{\mu}(s^{t-1})\right) \\
& + \Omega_1^L(s^{t-1}) \sum_{k=0}^{1} \mathbb{P}\left(k, 1, \iota^L(s^{t-1})\right) \cdot \mathbb{P}\left(k, 1, \Delta(s^{t-1})\right) \\
& + \sum_{n=2}^{\infty} \Omega_n^L(s^{t-1}) \cdot \sum_{k=0}^{1} \mathbb{P}\left(k, n, \iota^L(s^{t-1})\right) \cdot \mathbb{P}\left(k+n-1, n, \Delta(s^{t-1})\right)
\end{aligned} \tag{42}
$$

$$
\begin{aligned}
\Omega^{d}_{\tilde{n}>1}(s^{t}) = & \sum_{n=\mathbb{I}^{+}\left(\frac{\tilde{n}}{2}\right)}^{\tilde{n}-1} \Omega^{d}_{n}(s^{t-1}) \cdot \sum_{k=0}^{2n-\tilde{n}} \mathbb{P}\left(\tilde{n}-n+k, n, \iota^{d}(s^{t-1})\right) \cdot \mathbb{P}\left(k, n, \Delta(s^{t-1})\right) \\
& + \Omega^{d}_{\tilde{n}}(s^{t-1}) \sum_{k=0}^{\tilde{n}} \mathbb{P}\left(k, \tilde{n}, \iota^{d}(s^{t-1})\right) \cdot \mathbb{P}\left(k, \tilde{n}, \Delta(s^{t-1})\right) \\
& + \sum_{n=\tilde{n}+1}^{\infty} \Omega^{d}_{n}(s^{t-1}) \cdot \sum_{k=0}^{\tilde{n}} \mathbb{P}\left(k, n, \iota^{d}(s^{t-1})\right) \cdot \mathbb{P}\left(k+n-\tilde{n}, n, \Delta(s^{t-1})\right)
\end{aligned} \tag{43}
$$

where $\mathbb{I}^{+}(x)$ refers to the integer closest to x such that $\mathbb{I}^{+}(x) \geq x$. To understand the intuition of these expressions, we will first focus on the general expression for $\Omega^{d}_{\tilde{n}>1}(s^{t})$. The first line represents the successful innovators of lower-size classes that achieve size $\tilde{n}$, the second term represents the firms that keep their $\tilde{n}$ products, and the third term shows the formerly larger firms that shrink to exactly $\tilde{n}$ products. Further simplifications lead to

$$
\begin{aligned}
\Omega^{d}_{\tilde{n}>1}(s^{t}) = & \sum_{n=\mathbb{I}^{+}\left(\frac{\tilde{n}}{2}\right)}^{\tilde{n}} \Omega^{d}_{n}(s^{t-1}) \cdot \sum_{k=\tilde{n}-n}^{n} \mathbb{P}\left(k, n, \iota^{d}(s^{t-1})\right) \cdot \mathbb{P}\left(k-(\tilde{n}-n), n, \Delta(s^{t-1})\right) \\
& + \sum_{n=\tilde{n}+1}^{\infty} \Omega^{d}_{n}(s^{t-1}) \cdot \sum_{k=0}^{\tilde{n}} \mathbb{P}\left(k, n, \iota^{d}(s^{t-1})\right) \cdot \mathbb{P}\left(k-(\tilde{n}-n), n, \Delta(s^{t-1})\right)
\end{aligned} \tag{44}
$$

Because every product line belongs to a firm, we have

$$
\sum_{n=1}^{\infty} \left(\Omega^{H}_{n}(s^{t}) + \Omega^{L}_{n}(s^{t})\right) \cdot n = \Lambda. \tag{45}
$$

The total mass of firms in the economy is

$$
\Omega(s^{t}) = \sum_{n=1}^{\infty} \left(\Omega^{H}_{n}(s^{t}) + \Omega^{L}_{n}(s^{t})\right).
$$

In line with its empirical counterpart, the quarterly entry rate is defined as

$$
\text{Entry rate}\left(s^{t}\right) = \frac{2 \cdot M(s^{t-1})}{\Omega(s^{t}) + \Omega(s^{t-1})}.
$$

Analogously, the quarterly exit rate is given by

$$\text{Exit rate}\,(s^t) = \frac{2 \cdot \left[\sum_{d\in\{H,L\}} \sum_{n=1}^{\infty} \Omega_n^d(s^t) \cdot \mathbb{P}\left(0, n, \iota^d(s^{t-1})\right) \cdot \mathbb{P}\left(n, n, \Delta(s^{t-1})\right)\right]}{\Omega(s^t) + \Omega(s^{t-1})}$$
$$= \frac{2 \cdot \sum_{d\in\{H,L\}} \sum_{n=1}^{\infty} \Omega_n^d(s^t) \cdot \left[(1-\iota^d(s^{t-1}))\Delta(s^{t-1})\right]^n}{\Omega(s^t) + \Omega(s^{t-1})}.$$

Given an initial distribution and a sequences of innovation intensities and replacement rates, we can uniquely pin down the evolution of the size distribution in the economy. When comparing with data, we measure entry and exit annually.

1.3. Equilibrium Definition

To render the model stationary, we adopt the following convention: Any lowercase variable represents the productivity scaled version of its uppercase counterpart; for instance, the stationary transformation of output is given by $y(s^t) = \frac{Y(s^t)}{A(s^t)}$. In the case of capital and bonds, because of the timing convention, we have $k = \frac{K(s^{t-1})}{A(s^t)}$ and $b = \frac{B(s^{t-1})}{A(s^t)}$. This transformation is performed for consumption, bond holdings, capital, wages, intermediate goods production, investment, and output. With this transformation, we define a stationary competitive equilibrium for this economy:

Definition 1. *A competitive equilibrium for this small open economy, given an initial efficiency level $q_j(0)$ for every product line, an initial fraction of high-type incumbents, and initial levels of bond holding and capital for the household, is given by the following:*

1. *Household optimally chooses $\{c(s^t), b(s^t), k(s^t), l(s^t)\}$ given prices and transfers to solve* (14) *subject to* (9) *.*
2. *Final good producer optimally chooses $\left\{\{x_j(s^t)\}_{j\in[0,\Lambda]}, k(s^{t-1})\right\}$ given prices to solve* (15)*.*
3. *Intermediate firm f with n product lines of type d optimally chooses its price $\{p_j(s^t)\}_{j\in[0,\Lambda]}$ and its production and expansion labor usage $\left\{l_f^d(s^t,n) \equiv n \cdot l^d(s^t), l_{r,f}^d(s^t,n) \equiv n \cdot l_r^d(s^t)\right\}$ given wages and their type according to* (19)*,* (21)*, and* (23)*.*
4. *Financial intermediary optimally chooses $\{M(s^t)\}$ given values and prices in order to satisfy* (26)*.*

5. Capital markets clear in every history, and intermediate good markets clear in every history for every product line.

6. Labor, asset, and final good markets clear in every history:

$$l(s^t) = \Lambda\mu(s^t)\left(l^H(s^t)+l_r^H(s^t)\right)+\Lambda(1-\mu(s^t))\left(l^L(s^t)+l_r^L(s^t)\right)+\kappa M(s^t) \quad (46)$$

$$d(s^t) = b(s^{t-1})-\eta\frac{\alpha y(s^t)}{1+\eta(R(s^{t-1})-1)}-M(s^t)\kappa w(s^t)\eta \quad (47)$$

$$nx(s^t) = y(s^t)-c(s^t)-i(s^t)-\frac{\psi}{2}y(s^t)\left(\frac{b(s^t)}{y(s^t)}\left(1+a(s^t)\right)-\bar{b}\left(1+\bar{g}\right)\right)^2 \quad (48)$$

7. $\left\{v_j^d(n,s^t)=n\cdot\bar{v}^d(s^t)\,,q_j(s^t)\right\}_{j\in[0,\Lambda]\,,d\in\{L,H\}}$ *and* $\mu(s^t)$ *evolve according to* (24), (4), *and* (30)*.*

8. The mass of firms of type d *with* n *product lines evolves according to* (41), (42), *and* (44)*.*

9. Every product belongs to a firm so that (45) *holds.*

10. Transversality and non-negativity conditions are met.

We can also define a balanced growth path (BGP) for this economy as follows:

Definition 2. *A BGP is a non-stochastic (*$\sigma_R=\sigma_z=0$*) equilibrium where* $\{M(s^t)\}$ *is constant, and consumption, bond holdings, capital, wages, intermediate goods production, investment, net exports, and output grow at a constant rate. Along the BGP,* Ω_n^d *is constant for every* n *and* d*.*

1.4. Normalized System of Equations

1.4.1. Representative Household

$$m(s^{t+1}) = \frac{\beta}{(1+a(s^t))^{\gamma}} \frac{\left(c(s^{t+1}) - \Theta\left(l(s^{t+1})\right)^{\chi}\right)^{-\gamma}}{\left(c(s^t) - \Theta\left(l(s^t)\right)^{\chi}\right)^{-\gamma}} \tag{49}$$

$$b(s^t) = \left\{ \frac{\mathbb{E}\left[m(s^t, s_{t+1})|s^t\right] R(s^t) - 1}{\psi} + \bar{b}\left(1+\bar{g}\right) \right\} \frac{y(s^t)}{(1+a(s^t))} \tag{50}$$

$$1 = \mathbb{E}\left[m(s^{t+1}) \frac{r(s^{t+1}) + (1-\delta) - \frac{\phi}{2}\left([1+\bar{g}]^2 - \left[\frac{k(s^{t+1})}{k(s^t)}\left(1+a(s^{t+1})\right)\right]^2 \right)}{1 + \phi\left[\frac{k(s^t)}{k(s^{t-1})}\left(1+a(s^t)\right) - (1+\bar{g})\right]} |s^t \right] \tag{51}$$

$$l(s^t) = \left(\frac{w(s^t)}{\Theta\chi}\right)^{\frac{1}{\chi-1}} \tag{52}$$

$$i(s^t) = k(s^t)\left(1+a(s^t)\right) - (1-\delta)k(s^{t-1}) + \frac{\phi}{2}k(s^{t-1})\left(\frac{k(s^t)}{k(s^{t-1})}(1+a(s^t)) - (1+\bar{g})\right)^2 \tag{53}$$

$$\begin{aligned} c(s^t) = {} & w(s^t)l(s^t) + r(s^t)k(s^{t-1}) + b(s^{t-1})R(s^{t-1}) + t(s^t) - i(s^t) - b(s^t)\left(1+a(s^t)\right) \\ & - \frac{\psi}{2}y(s^t)\left(\frac{b(s^t)}{y(s^t)}(1+a(s^t)) - \bar{b}(1+\bar{g})\right)^2 \end{aligned} \tag{54}$$

1.4.2. Final Good Producer

$$y(s^t) = \exp\left(z(s^t)\right) \cdot \left(\left(l^H(s^t)\right)^{\mu(s^t)} \left(l^L(s^t)\right)^{1-\mu(s^t)}\right)^{\alpha} \left(k(s^{t-1})\right)^{1-\alpha} \tag{55}$$

$$k(s^{t-1}) = \frac{(1-\alpha)y(s^t)}{r(s^t)} \tag{56}$$

1.4.3. Intermediate Good Producers

$$l^d(s^t) = \frac{\frac{\alpha}{\Lambda} y(s^t)}{w(s^t)(1+\sigma^d)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)} \tag{57}$$

$$\pi_j^d(s^t) = \frac{\alpha}{\Lambda} \frac{\sigma^d}{(1+\sigma^d)} y(s^t) \tag{58}$$

$$\begin{aligned} \bar{v}^d(s^t) &= \pi^d(s^t) - w(s^t)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)\varphi l^d(s^t)^\xi \\ &\quad + \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\left(1-\Delta(s^t)+\iota^d(s^t)\right)\bar{v}^d(s^{t+1})|s^t\right] \end{aligned} \tag{59}$$

$$\iota^d(s^t) = \left(\frac{\mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\bar{v}^d(s^{t+1})|s^t\right]}{\varphi\xi w(s^t)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)}\right)^{\frac{1}{\xi-1}} \tag{60}$$

$$l_r^d(s^t) = \varphi\left(\iota^d(s^t)\right)^\xi \tag{61}$$

1.4.4. Financial Intermediary

$$M(s^t) = 1 - \left[\frac{\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)w(s^t)\kappa - \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\bar{v}^L(s^{t+1})|s^t\right]}{\mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\left(\bar{v}^H(s^{t+1})-\bar{v}^L(s^{t+1})\right)|s^t\right]}\right]^{\frac{1}{\nu}} \tag{62}$$

$$\tilde{\mu}(s^t) = \frac{1}{\nu+1}\left[\frac{1-\left[1-M(s^t)\right]^{\nu+1}}{M(s^t)}\right] \tag{63}$$

1.4.5. Aggregate Variables

$$a(s^t) = \left[\left(1+\sigma^H\right)^{\tilde{\mu}(s^t)}\left(1+\sigma^L\right)^{1-\tilde{\mu}(s^t)}\right]^{\frac{M(s^t)}{\Lambda}}\left(1+\sigma^H\right)^{\mu(s^t)\iota^H(s^t)}\left(1+\sigma^L\right)^{\left(1-\mu(s^t)\right)\iota^L(s^t)} - 1 \tag{64}$$

$$\begin{aligned}
\mu(s^t) &= \mu(s^{t-1}) + \frac{M(s^{t-1})}{\Lambda}\left[\tilde{\mu}(M(s^{t-1})) - \mu(s^{t-1})\right] \\
&\quad + \mu(s^{t-1})\left(1-\mu(s^{t-1})\right)\left(\iota^H(s^{t-1}) - \iota^L(s^{t-1})\right) && (65) \\
\Delta(s^t) &= \frac{M(s^t)}{\Lambda} + \mu(s^t)\iota^H(s^t) + \left(1-\mu(s^t)\right)\iota^L(s^t) && (66) \\
t(s^t) &= \mu(s^t)\Lambda\left[\pi^H(s^t) - \left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)w(s^t)l_r^H(s^t)\right] \\
&\quad + \left(1-\mu(s^t)\right)\Lambda\left[\pi^L(s^t) - \left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)w(s^t)l_r^L(s^t)\right] \\
&\quad - \left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)M(s^t)\kappa w(s^t) && (67) \\
nx(s^t) &= y(s^t) - c(s^t) - i(s^t) - \frac{\psi}{2}y(s^t)\left(\frac{b(s^t)}{y(s^t)}(1+a(s^t)) - \bar{b}(1+\bar{g})\right)^2 && (68) \\
d(s^t) &= b(s^{t-1}) - \eta w(s^t)l(s^t) && (69) \\
l(s^t) &= \Lambda\mu(s^t)\left(l^H(s^t) + l_r^H(s^t)\right) + \Lambda(1-\mu(s^t))\left(l^L(s^t) + l_r^L(s^t)\right) + \kappa M(s^t) && (70)
\end{aligned}$$

1.4.6. Exogenous Shocks

$$\ln\left(\frac{R(s^t)}{\bar{R}}\right) = \rho_R \ln\left(\frac{R(s^{t-1})}{\bar{R}}\right) + \sigma_R\epsilon_{R,t} \quad \text{where} \quad \epsilon_{R,t} \overset{iid}{\sim} N(0,1), \tag{71}$$

$$z(s^t) = \rho_z z(s^{t-1}) + \sigma_z\epsilon_z(s^t) \quad \text{where} \quad \epsilon_z(s^t) \overset{iid}{\sim} N(0,1) \tag{72}$$

1.5. Solving for Balanced Growth Path

Consider a system with three equations and three unknowns $(\iota^H, \iota^L, \bar{z})$ that characterizes the BGP of this economy. We start with some auxiliary equations. After imposing

BGP, the composition of entrants and incumbents are given by

$$\tilde{\mu} = \frac{1}{\nu+1}\left[\frac{1-(1-M)^{\nu+1}}{M}\right]$$
$$\mu = \frac{\iota^H - \iota^L - \frac{M}{\Lambda} + \sqrt{\left(\iota^H - \iota^L - \frac{M}{\Lambda}\right)^2 + 4\tilde{\mu}\frac{M}{\Lambda}\left(\iota^H - \iota^L\right)}}{2\left(\iota^H - \iota^L\right)}.$$

Therefore, the replacement rate of the economy is given by

$$\Delta = \frac{M}{\Lambda} + \mu\iota^H + (1-\mu)\,\iota^L.$$

The long-run growth rate of the economy can be characterized as

$$a = \left[\left(1+\sigma^H\right)^{\tilde{\mu}}\left(1+\sigma^L\right)^{1-\tilde{\mu}}\right]^{\frac{M}{\Lambda}}\left(1+\sigma^H\right)^{\mu\iota^H}\left(1+\sigma^L\right)^{(1-\mu)\iota^L} - 1.$$

From (49) and (50), in order to have $\frac{b}{y} = \bar{b}$ so that no bond-holding costs are paid in the long-run, there is a unique value for the internal calibration of β:

$$\beta = \frac{(1+\bar{g})^\gamma}{\bar{R}}.$$

The normalized long-run level of capital is given by

$$k = \frac{1-\alpha}{\bar{R}-1+\delta}y.$$

The demand for labor at the product line level is given by

$$l^d = \frac{\frac{\alpha}{\Lambda}y}{w(1+\sigma^d)\left(1+\eta\left(\bar{R}-1\right)\right)}.$$

Replacing the capital demand for the final good producer and the demand for labor of the intermediate good producer in equation (55), we get the equilibrium wage:

$$w = \left(\frac{1-\alpha}{\bar{R}-(1-\delta)}\right)^{\frac{1-\alpha}{\alpha}}\frac{\frac{\alpha}{\Lambda}}{(1+\sigma^H)^{\mu}\left(1+\sigma^L\right)^{1-\mu}\left(1+\eta\left(\bar{R}-1\right)\right)}.$$

We can characterize output using the labor market clearing condition from equation (70):

$$y = w\left(\frac{1+\eta\left(\bar{R}-1\right)}{\alpha}\right)\left(\frac{\left(\frac{w}{\Theta\chi}\right)^{\frac{1}{\chi-1}} - \Lambda\varphi\left[\mu\left(\iota^{H}\right)^{\xi} + (1-\mu)\left(\iota^{L}\right)^{\xi}\right] - \kappa M}{\frac{\mu}{1+\sigma^{H}} + \frac{1-\mu}{1+\sigma^{L}}}\right).$$

We now characterize the profits associated to product lines and the value of firms:

$$\begin{aligned}
\pi^{d} &= \frac{\alpha}{\Lambda}\left(\frac{\sigma^{d}}{1+\sigma^{d}}\right)y \\
\bar{v}^{d} &= \frac{\pi^{d} - w\left(1+\eta\left(\bar{R}-1\right)\right)\varphi\left(\iota^{d}\right)^{\xi}}{1-\frac{1+\bar{g}}{\bar{R}}\left(1+\iota^{d}-\Delta\right)}.
\end{aligned}$$

The BGP is the solution of the following nonlinear system of three equations and three unknowns:

$$\begin{aligned}
\iota^{d} &= \left(\frac{(1+\bar{g})\bar{v}^{d}}{\bar{R}\varphi\xi w\left(1+\eta\left(\bar{R}-1\right)\right)}\right)^{\frac{1}{\xi-1}} \\
(1-M)^{\nu} &= \frac{(1+\eta(\bar{R}-1))w\kappa - \frac{1+\bar{g}}{\bar{R}}\bar{v}^{L}}{\frac{1+\bar{g}}{\bar{R}}\left(\bar{v}^{H}-\bar{v}^{L}\right)}.
\end{aligned}$$

With the solution of this system, we can characterize every variable of the BGP. In particular, the household budget constraint pins down c, and nx is determined by the final good market clearing. Importantly, all of the above derivations are independent from the size distribution of firms. Nevertheless, the next section shows that the firm size distribution is unique and well defined.

1.5.1. Long-Run Distribution: Poisson Case

In continuous time, with Poisson processes, we can find an analytic expression for the distribution of firms. Because we will use this distribution as a guess in the algorithm for the binomial case, we first characterize this distribution. Recall that entry rate equals exit rate along the BGP. Therefore, the total mass of firms with one product line is the following:

$$\Omega_1^{H} + \Omega_1^{L} = \frac{M}{\Delta}.$$

Along the BGP we have

$$\Omega_2^H = \frac{\Omega_1^H \left(\Delta + \iota^H\right) - M\tilde{\mu}}{2\Delta}$$
$$\Omega_2^L = \frac{\Omega_1^L \left(\Delta + \iota^L\right) - M\left(1 - \tilde{\mu}\right)}{2\Delta}$$
$$\Omega_{n+1}^d = \frac{n\Omega_n^d \left(\Delta + \iota^d\right) - (n-1)\iota^d \Omega_{n-1}^d}{(n+1)\Delta}, \quad \forall d \in \{H, L\}, n > 2.$$

We also have

$$\begin{aligned}
\mu &= \frac{1}{\Lambda}\sum_{n=1}^{\infty} n \cdot \Omega_n^H \\
&= \frac{1}{\Lambda}\left[\Omega_1^H + \frac{\Omega_1^H\left(\Delta + \iota^H\right) - M\tilde{\mu} + \sum_{n=2}^{\infty}\left[n\Omega_n^H\left(\Delta + \iota^H\right) - (n-1)\iota^H\Omega_{n-1}^H\right]}{\Delta}\right] \\
&= \frac{1}{\Lambda}\left[\Omega_1^H + \frac{\Omega_1^H\Delta - M\tilde{\mu} + \sum_{n=2}^{\infty} n\Omega_n^H\Delta}{\Delta}\right] \\
&= \frac{1}{\Lambda}\left[\Omega_1^H - \frac{M\tilde{\mu}}{\Delta} + \sum_{n=1}^{\infty} n\Omega_n^H\right] \quad \Rightarrow \quad \Omega_1^H = \frac{M\tilde{\mu}}{\Delta} \\
&\Rightarrow \Omega_1^L = \frac{M\left(1 - \tilde{\mu}\right)}{\Delta}.
\end{aligned}$$

Thus, we can solve for all the shares as

$$\begin{aligned}
\Omega_2^H &= \frac{\Omega_1^H}{2}\left(\frac{\iota^H}{\Delta}\right) \quad \text{and} \quad \Omega_2^L = \frac{\Omega_1^L}{2}\left(\frac{\iota^L}{\Delta}\right) \\
\Omega_3^H &= \frac{\Omega_1^H}{3}\left(\frac{\iota^H}{\Delta}\right)^2 \quad \text{and} \quad \Omega_3^L = \frac{\Omega_1^L}{3}\left(\frac{\iota^L}{\Delta}\right)^2 \\
&\dots \\
\Omega_n^d &= \frac{\Omega_1^d}{n}\left(\frac{\iota^d}{\Delta}\right)^{n-1}.
\end{aligned}$$

The number of firms in each size class decreases with the number of product lines. Nevertheless, the number of high types decreases at a slower rate. Therefore, the share of high-type firms increases with the number of product lines:

$$\frac{\Omega_n^H}{\Omega_n^L} = \frac{\tilde{\mu}}{1 - \tilde{\mu}}\left(\frac{\iota^H}{\iota^L}\right)^{n-1}.$$

The total mass of firms in the economy of each type is given by

$$\Omega^d = \Omega_1^d \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\iota^d}{\Delta} \right)^{n-1} = \frac{\Delta}{\iota^d} \ln \left(\frac{\Delta}{\Delta - \iota^d} \right) \Omega_1^d.$$

Therefore, the long-run size distribution by type is logarithmic:

$$P_n^d = \frac{\left(\frac{\iota^d}{\Delta} \right)^n}{n \ln \left(\frac{\Delta}{\Delta - \iota^d} \right)},$$

The unconditional size distribution is characterized by

$$P_n = \frac{\frac{\Omega_1^H}{n} \left(\frac{\iota^H}{\Delta} \right)^{n-1} + \frac{\Omega_1^L}{n} \left(\frac{\iota^L}{\Delta} \right)^{n-1}}{\frac{\Delta}{\iota^H} \ln \left(\frac{\Delta}{\Delta - \iota^H} \right) \Omega_1^H + \frac{\Delta}{\iota^L} \ln \left(\frac{\Delta}{\Delta - \iota^L} \right) \Omega_1^L} = \frac{\tilde{\mu} \left(\frac{\iota^H}{\Delta} \right)^n + (1 - \tilde{\mu}) \left(\frac{\iota^L}{\Delta} \right)^n \frac{\iota^H}{\iota^L}}{n \left[\tilde{\mu} \ln \left(\frac{\Delta}{\Delta - \iota^H} \right) + (1 - \tilde{\mu}) \ln \left(\frac{\Delta}{\Delta - \iota^L} \right) \frac{\iota^H}{\iota^L} \right]}.$$

1.5.2. Long-Run Distribution: Binomial Case

Algorithm:

1. Use Poisson as initial guess for Ω_n^d for $n = 1...\bar{n}$.
2. Iterate using the law of motion and the BGP values until $\max \left(\Omega_n^d(s^t) - \Omega_n^d(s^{t+1}) \right) <$ tol.

1.6. Mass and Composition Effects during a Crisis

Consider first a simple version of the model where there is no incumbent innovation and all growth is due to entrants. Moreover, the entry loss has no dynamic impact on productivity, as there is no innovation by incumbents. In this case, aggregate productivity growth boils down to

$$1 + a(s_t) \equiv \frac{A(s^t, s_{t+1})}{A(s^t)} = \left[\left(1 + \sigma^H \right)^{\tilde{\mu}(s^t)} \left(1 + \sigma^L \right)^{1 - \tilde{\mu}(s^t)} \right]^{\frac{M(s^t)}{\Lambda}}.$$

Applying logs obtains

$$
\begin{aligned}
\ln 1 + a(s_t) &= \frac{M(s^t)}{\Lambda}\left(\tilde{\mu}(s^t)\ln\left(1+\sigma^H\right) + \left(1-\tilde{\mu}(s^t)\right)\ln\left(1+\sigma^L\right)\right) \\
\frac{\partial \ln 1 + a(s_t)}{M(s^t)} &= \frac{\tilde{\mu}(s^t)\ln\left(1+\sigma^H\right) + (1-\tilde{\mu}(s^t))\ln\left(1+\sigma^L\right)}{\Lambda} + \frac{M(s^t)}{\Lambda}\frac{\partial\tilde{\mu}(s^t)}{\partial M(s^t)}\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right) \\
&= \frac{1}{\Lambda}\left\{\ln\left(1+\sigma^L\right) + \left[\tilde{\mu}(s^t) + M(s^t)\frac{\partial\tilde{\mu}(s^t)}{\partial M(s^t)}\right]\ln\left(\frac{1+\sigma^H}{1+\sigma^L}\right)\right\}.
\end{aligned}
$$

If the term in squared bracket is positive, then a decrease in entry always decreases productivity growth. The second term inside the brackets represents the composition effect in this simple case. Some algebra yields

$$
\tilde{\mu} + M\frac{\partial\tilde{\mu}}{\partial M} = (1-M)^{\nu} > 0.
$$

Therefore, in this simpler case, the composition effect is always dominated by the mass effect in terms of productivity.

The case with incumbents' innovation is more complicated, as it included two new forces. First, incumbents' innovation also reacts to the crisis. In fact, ι^H and ι^L also respond to the crisis. Second, the increase in composition by entrants now has a dynamic effect as those firms will innovate at a higher rate. The full expression becomes

$$
\begin{aligned}
1 + a(s_t) \equiv \frac{A(s^t, s_{t+1})}{A(s^t)} = & \underbrace{\left[\left(1+\sigma^H\right)^{\tilde{\mu}(s^t)}\left(1+\sigma^L\right)^{1-\tilde{\mu}(s^t)}\right]^{\frac{M(s^t)}{\Lambda}}}_{\text{Entrants}} \\
& \times \left[\underbrace{\left(1+\sigma^H\right)^{\mu(s^t)\iota^H(s^t)}\left(1+\sigma^L\right)^{\left(1-\mu(s^t)\right)\iota^L(s^t)}}_{\text{Incumbents}}\right].
\end{aligned}
$$

The innovation of incumbents, ι^d, is affected by two forces during the crisis. First, the crisis decreases aggregate creative destruction Δ by reducing aggregate innovation in the economy. This would imply a cleansing effect of crisis that increases productivity growth. Second, even with a lower Δ, the crisis decreases firm values (and then the return to innovation) by increasing the stochastic discount rate. This last force dominates in every quantitative experiment we have performed. In fact, ι^H and ι^L always decrease during crises. The second effect comes from the fact that higher $\tilde{\mu}(s^t)$ imply a persistently higher $\mu(s^t)$ after the crisis. This can fuel the economy given that $\iota^H > \iota^L$. While it is not straightforward to establish theoretically whether higher $\mu(s^t)$ could dominate the other effects, quantitatively, we observe in all calibrations we have explored that the first-order effect is the drop in incumbents' innovation rates. Therefore, the overall effect of a crisis is again negative, and the rise in $\mu(s^t)$—the composition effect—just softens the effect of the crisis and increases the speed of the recovery.

1.7. Long-Run Cost and Endogenous Growth

Growing variables, such as output or investment, are normalized by A_t. Denoting log-deviations of a variable H from its last period value by a hat ($\hat{H}_t = \ln(H_t/H_{t-1})$), we will now focus on output to highlight the source of the long-run cost:

$$y_t = \frac{Y_t}{A_t} \Rightarrow \hat{Y}_t \approx \hat{y}_t + \hat{A}_t. \tag{73}$$

In the absence of a shock, because y_t is constant, we get $\hat{Y}_t = \hat{A}_t \approx a_{ss}$. Hence, for scaled variables, we can define the distance at time t between the nonshocked economy and the one subject to the shock as $\tilde{x}_t^Y$:

$$\tilde{x}_t^Y \approx \sum_{i=1}^{i=t} \left\{ \hat{y}_i + \hat{A}_i \right\} - t * a_{ss}. \tag{74}$$

The main difference between models with exogenous growth and models with endogenous growth is that, because growth is exogenous, $\hat{A}_t \approx a_{ss}$, and then $\tilde{x}_t^Y = \sum_{i=1}^{i=t} \hat{y}_i$. Because y_t is stationary, this term converges to zero when time goes to infinity. This convergence illustrates why there is no long-run cost of a sudden stop for a model with exogenous growth. But a model with endogenous growth has a long-run cost (LRC), in any normalized variable, approximately equal to

$$LRC \approx \lim_{t\to\infty} \left\{ t * a_{ss} - \sum_{i=1}^{i=t} \left\{ \hat{A}_i \right\} \right\} < \infty. \tag{75}$$

Because $\hat{A}_t$ converges to a_{ss}, this long-run cost is finite. Moreover, as is clear from equation (74), this long-run cost arises only for variables that exhibit long-run growth.

2. Empirical Analysis

2.1. ENIA: Data Cleaning

The Encuesta Nacional Industrial Anual (ENIA, Annual National Industrial Survey) conducted the by the INE covers all manufacturing plants in Chile with more than 10 workers. Our version extends from 1995 to 2007.

We eliminate observations with one or more of the following inconsistencies, with original variable names provided in parenthesis: negative electricity consumption (*elecons*), worked days less than or equal to 0 (*diatra*), gross value of the production less than value added (*vpn*<*va*), value added less than 0 (*va*), remuneration of workers equal to 0 (*rem-pag*), size equal to 0 (*tamano*), ISIC code less than 3000 (bad coding in *sector*), and sales income less than income from exports (*ingtot*<*ingexp*). Finally, we include in the analysis 22 of the 29 three-digit industries. We exclude commodity-related industries (353 and 354 for petroleum, 371 and 372 for metals). We also drop industries where revenue productivity cannot be reliable estimated (one or the sum of the input elasticities are outside the unit circle, typically due to the lack of observations); this is the case for 361 (pottery), 323 (leather), and 314 (tobacco). After all the cleaning procedures, the sample has 85% of the firms-year observations and 90% of the workers. The most important drop is copper related (371 and 372), implying a combined loss of 2.3% of observations and 5.6% of workers).

2.2. Variable Construction and Other Controls

We calculate entry rates at year t at the industry level for each cohort, dividing the number of new plants in year t by the average of the total plants in years t and $t-1$. The variable used to build the productivity used in Table II is value added. We define capital as the end-of-period value of land, machinery, buildings and vehicles (*salter+salmaq+saledi+salveh*). We deflate monetary variables using three-digit industry level deflators provided by the INE. The revenue (*ingtot-revval-reviva*) used to calculate the Herfindahl-Hirschman concentration index (HHI) excludes nonmanufactured products (reselling products and their tax shield); the costs include wages and exclude the costs and taxes associated with non-manufactured products (*costot-mrevval-mreviva+rempag*). The index of manufacturing production (22866EY.ZF...), the unemployment rate (22867R..ZF...), and the producer price and wholesale price index (PPI/WPI, 22863...ZF...) are taken from the IFS database. The labor cost index is from the Chilean central bank.

For each three-digit industry (denoted by s) we separately estimate the following production function:

$$\log y_{it} = d_t^s + \beta^{sl}\log l_{it} + \beta^{sk}\log k_{it} + \log z_{it} + \varepsilon_{it},$$

where y_{it} is real value added for firm i in year t, d_t^s is a time-fixed effect, l_{it} is total workers, and k_{it} is real capital stock. The coefficient β^{sl} denotes the industry-specific elasticity of value added with respect to labor and β^{sk} denotes the elasticity of value added with respect to capital. We estimate these elasticities using the methodology described in Wooldridge (2009). Using the estimated elasticities $\hat{\beta}^{sl}$ and $\hat{\beta}^{sk}$, we calculate firm productivity as:

$$\log z_{it} = \log y_{it} - \hat{\beta}^{sl}\log l_{it} - \hat{\beta}^{sk}\log k_{it}$$

Table **??** shows the estimated elasticities. The sum of the elasticities is always less than one.

2.3. Macro Data

In this subsection, we present the sources of the macroeconomic data used in this paper and the behavior of the aggregated time series during the crisis. To start, Chile is a small economy both in terms of population and aggregate output. It has also experienced spectacular growth, which led it to be the first OECD member in South America (2010). Its trade and debt ratio justify the small open economy framework adopted in this paper. In particular, while its trade-to-GDP ratio is quite high, according to the *World Trade Organization* database, in 2011 Chile had 0.45% of the world's exports and 0.41% of the world's imports. Chile is also the 7^{th} freest economy in the world (2013 *International Economic Freedom Ranking*).

The main source of data for the quantitative analysis in Section 5 is the Central Bank of Chile, from whose database we obtained real GDP, real gross fixed capital information, and real consumption series. In order to be able to cover pre-crisis years, we used the versions in millions of 2003 Chilean Pesos, spanning between 1996:Q1 and 2011:Q2. To conduct the empirical analysis in 4, we used additional data from the International Finance Statistics (IFS) database from the International Monetary Fund (IMF). From that source, we obtained exchange rate (228..RF.ZF...), financial accounts (22878BJ DZF...), direct investment abroad (22878BDDZF...), direct investment in Chile (22878BEDZF...), net errors and omissions (22878CADZF...), exports (22890C..ZF...), and imports (22898C..ZF...). We use employment data from the Instituto Nacional de Estadística (INE, National institute of Statistics) of Chile and hours worked per week from the *Encuesta de Ocupación y Desocupación* from the Economics Department of *Universidad de Chile*. The interest rate is the average observed real interest rate for commercial loans with a maturity of 3 to 12 months; this data is provided by the Chilean Central Bank online.

2.4. Working Capital in the Data

In order to discipline the working capital parameter in the model, we use firm-level information on interest payments (*intgas*) and total cost of production (*totcost*) from ENIA. We link these variables to their model counterparts using the following relationship:

$$\eta\left(R(s^{t-1})-1\right)(\text{production cost}) = \text{interest spending} \Rightarrow$$
$$\eta = \frac{\text{interest spending}}{(\text{production cost})\left(R(s^{t-1})-1\right)},$$

where R is the Chilean real interest rate. We derive this ratio at the firm level. The value of η is roughly 50% before the crisis period when calculated as the simple average across firms. When firm-specific values for η are weighted by the employment size of firms, the average value increases to 70% for the same period. Taking an average value of these two estimates, we use $\eta = 60\%$ in our baseline calibration. Online Appendix 3.2.1 presents a robustness analysis for different values of η.

2.5. Hausman and Taylor (1981)

The method can be summarized as a four-step procedure. First, a fixed-effects regression delivers consistent estimators, $\hat{\beta}_1$ and $\hat{\beta}_2$, that are used to retrieve estimators $\hat{u}_{i,t}$ and $\hat{\sigma}_u$. The second step is an instrumental variables (IV) regression with $\hat{u}_{i,t}$ as dependent variable,

Z^1 and Z^2 as independent variables, and Z^1 and X^1 as instruments; this delivers a consistent estimator for $\tilde{\sigma}$ (the dispersion of the residual). Third, an estimator for the variance of the unobserved fixed-effect component can be built as $\hat{\sigma}_\mu^2 = \tilde{\sigma}^2 - \frac{\hat{\sigma}_u^2}{T}$, in order to form the usual generalized least squares (GLS) correction. Finally, the GLS correction is used to transform the original equation and estimate all the coefficients simultaneously in equation (37), using an IV procedure where the instruments are given by Z^1, the mean of X^1, and the deviations from the mean of X^1 and X^2. After every estimation, we perform the Sargan-Hansen test to assess the validity of the instrumental variables procedure.

Table X presents the details of the regression results from the main text. In our regressions we use as time-variant exogenous variables ($X_{i,t}^1$) four macroeconomic aggregates: an index of manufacturing production, the unemployment rate, an index of wholesale producer prices, and an index of the cost of labor.[49] The coefficients associated with these variables are stable across the productivity regressions. The signs of the significant coefficients suggest that productivity is higher when production is high, and inflation in producer prices are low. Higher labor cost are associated with higher productivity (this could be due to a selection effect, given that more productive firms can afford higher labor costs). There are four potentially endogenous post-entry controls ($X_{i,t}^2$): electricity consumption, number of workers, capital stock, and the age of the plant. We use five geographic regions and two-digit industry controls as time-invariant exogenous variables (Z_i^1). Besides the coefficients of interest, we include the initial capital stock of the plant. To control for competition at the moment of entry, we also include the Herfindahl-Hirschman concentration index of the industry at the particular region in the year of entry among the time-invariant endogenous variables (Z_i^2). In line with the firm dynamics literature, larger entrants are more profitable and more productive than smaller entrants.

[49] Because this method relies on $X_{i,t}^1$ to build instruments, and because they are all aggregate variables, we cannot include year dummies, which are perfectly correlated with our instruments.

Table X
Hausman and Taylor

	(1) $A_{i,t}$	(2) $A_{i,t}$	(3) $A_{i,t}$	(4) $A_{i,t}$
During Crisis	0.637***	0.618**		0.586**
	(0.227)	(0.275)		(0.245)
After Crisis		0.0893		0.0603
		(0.143)		(0.144)
Avg entry$_{j,0}$			-5.923***	
			(1.552)	
log Manu Prod$_t$	1.251***	1.250***	1.224***	1.090***
	(0.126)	(0.126)	(0.126)	(0.130)
Unemp Rate$_t$	-0.196	-0.198	-0.207	-0.190
	(0.556)	(0.557)	(0.555)	(0.488)
log PPI/WPI$_t$	-2.360***	-2.358***	-2.353***	-2.356***
	(0.165)	(0.165)	(0.165)	(0.164)
log L Cost$_t$	3.127***	3.099***	2.923***	3.258***
	(0.646)	(0.648)	(0.639)	(0.671)
log age$_{i,t}$	0.00636	0.00798	0.0224	0.0139
	(0.0219)	(0.0217)	(0.0210)	(0.0208)
HHI$_{r,j,0}$	27.21**	19.86	9.870	26.76
	(13.30)	(21.26)	(11.22)	(20.86)
log K$_{i,t_0}$	0.796***	0.726***	0.561***	0.726***
	(0.136)	(0.190)	(0.0657)	(0.206)
log Elec Con$_{i,t}$				0.0462***
				(0.00790)
log L$_{i,t}$				0.0638**
				(0.0258)
log K$_{i,t}$				-0.0576***
				(0.00943)
Ind. Control	Yes	Yes	Yes	Yes
Region Control	Yes	Yes	Yes	Yes
Constant	Yes	Yes	Yes	Yes
Observations	17646	17646	17646	17484
Sargan-Hansen (p)	0.495	0.242	0.0137	0.205

Standard errors in parentheses (bootstrapped (250), clustered by firm)
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

2.6. Cox Estimation

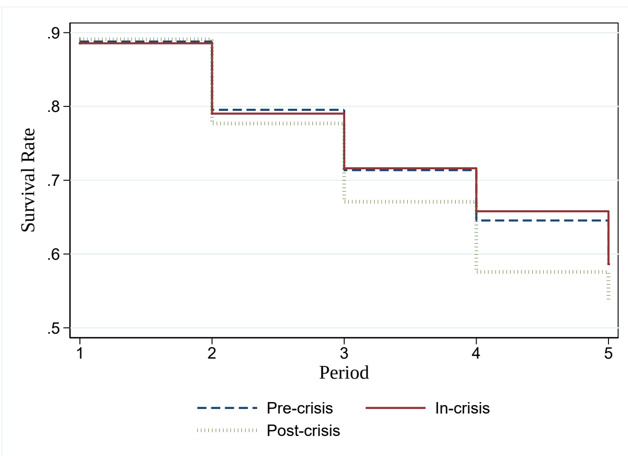
This section shows that the higher profitability of the cohorts born during the sudden stop is not due to *ex-post* selection. In particular, we perform the following stratified proportional hazard estimation to show that firms born during the crisis are not more likely to die at any horizon:

$$h_{r,c}\left(t|\boldsymbol{X}_i\right) \quad = \quad h_{0,r,c}\left(t\right)\exp\left[\boldsymbol{X}_i \cdot \boldsymbol{\beta}\right].$$

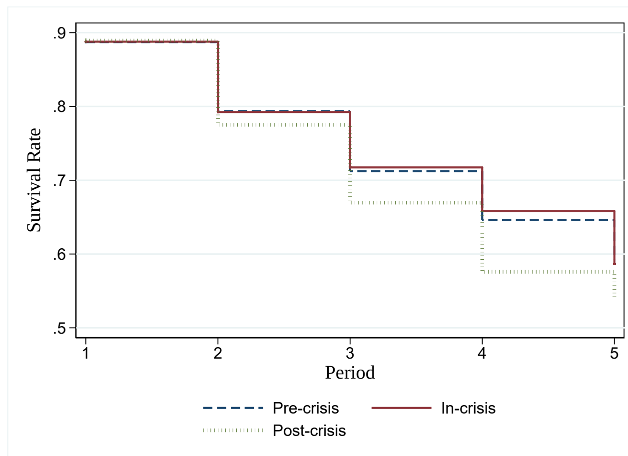
The two strata are geographical region (r) and time period (c). This means that the baseline hazard $h_{r,c}$ varies across these two dimensions. We divide Chile into five geographical regions. The time periods correspond to the *pre-crisis*, *crisis*, and *post-crisis* period of the second specification in the Hausman and Taylor estimation of Section 4. The Cox-Snell test cannot reject the proportional hazard structure with 95% confidence. Sub-index t refers to time, while i refers to a plant and j to an industry. Table XI shows the estimates of the common covariates.

Bigger plants have less probability of exiting (for both electricity consumption and number of workers), while the initial size increases the probability of exiting (for number of workers and electricity consumption). The specification controls for the industry cycle (using the average profitability of the industry $\bar{P}_{j,t}$ or the average productivity $\bar{A}_{j,t}$) and industry-specific effects. Figure XIII plots the survival rates at different horizons for cohorts born during the three different time periods in the central zone of Chile. We pick this zone because it concentrates most of the plants in the sample; the main message does not change when considering the other four regions.

Importantly, firms born during the crisis do not exit more than other cohorts. Moreover, they even seem stronger in this dimension in that, until year 6, they have a higher predicted survival probability than firms born either before or after the episode. Hence, *ex-post* selection does not explain the higher profitability of cohorts born during the sudden stop.



(A) Profitability



(B) Productivity

Figure XIII
Survival Rates, Cox Proportional Hazard Model

Notes: The figures plot average survival rates at different horizons for cohorts born during three different time periods in the central zone of Chile. The survival rates are estimated by a Cox proportional hazard model. The left panel is based on a model that uses profitability as an explanatory variable, whereas the estimated model for the right panel includes productivity.

Table XI
Proportional Hazard

	(1)	(2)
$\ln(L_{i,t})$	-0.631***	-0.628***
	(0.0713)	(0.0713)
$\ln(L_{i,0})$	0.548***	0.548***
	(0.0721)	(0.0721)
$\ln(elec_{i,t})$	-0.0742***	-0.0761***
	(0.0266)	(0.0267)
$\ln(elec_{i,0})$	0.0546**	0.0549**
	(0.0253)	(0.0254)
$\ln(K_{i,t})$	-0.0338	-0.0328
	(0.0249)	(0.0249)
$\ln(K_{i,0})$	-0.0295	-0.0304
	(0.0238)	(0.0238)
$P_{j,t}$	0.342	
	(0.261)	
$A_{j,t}$		0.185**
		(0.0861)
$HHI_{j,t}$	0.110	0.131
	(0.368)	(0.367)
Ind. Control	Yes	Yes
Observations	15149	15149
Plants	2981	2981
Exits	1758	1758
Hazard assumption test (p)	0.366	0.371

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3. Quantitative Analysis

3.1. Business Cycle Analysis

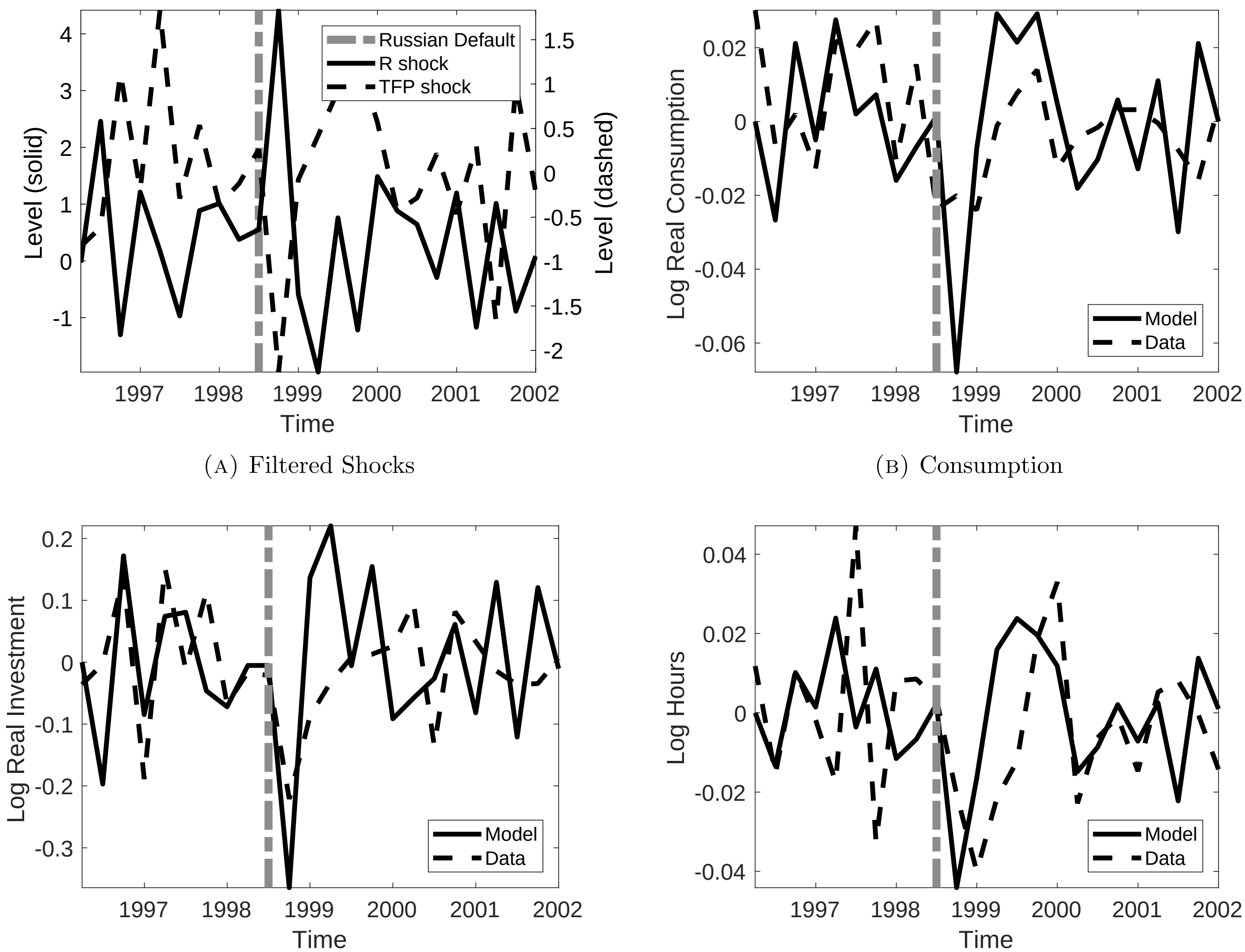
The Chilean crisis is characterized by an increase of 80 basis points in the quarterly interest rate between the beginning of the Asian crisis and the Russian default as well as by 4.5% drop in quarterly output. Using these series, we smooth out the interest and productivity innovations, and Figure XIVa shows these filtered series, which we then use to mimic the crisis in the model. The sudden stop is explained by a negative exogenous productivity shock and a simultaneous positive interest rate shock.

Feeding the filtered innovations into the model allows us to use the Chilean crisis to evaluate the business cycle behavior of the model. Figure XIV compares the model-implied path for the log differences of consumption, investment, and hours with data counterparts. The graphs establish that the model tracks well the behavior of the macro aggregates during the period.

Table XII shows the variance decomposition of the macro aggregates. The calibrated model is consistent with the evidence in Neumeyer and Perri (2005) and Uribe and Yue (2006), where interest rate fluctuations explain one-third of the fluctuations in Argentinian output. Because Chilean spreads are less volatile than Argentinian spreads, it is natural that interest rate fluctuations play a lower role with respect to Chilean output.

Figure XV shows the impulse response functions to a one standard deviation shock for the main macro variables.

As illustrated in figures XVa and XVb, the responses of output, labor, consumption and investment (right axis) are aligned with the literature. Consumption responds more on impact to interest rate shocks than output, but output responds more than consumption to stationary productivity shocks. As such, with a more volatile interest rate, the model would generate less smoothing in consumption. Importantly, none of the variables will return to its original long-run trend. In fact, for the case of the interest rate shock, the new path for these variables is permanently 0.1% lower. This hysteresis arises because of the permanent loss in the level of productivity, as shown in Figure XVa.



(A) Filtered Shocks

(B) Consumption

(C) Investment

(D) Hours

Figure XIV

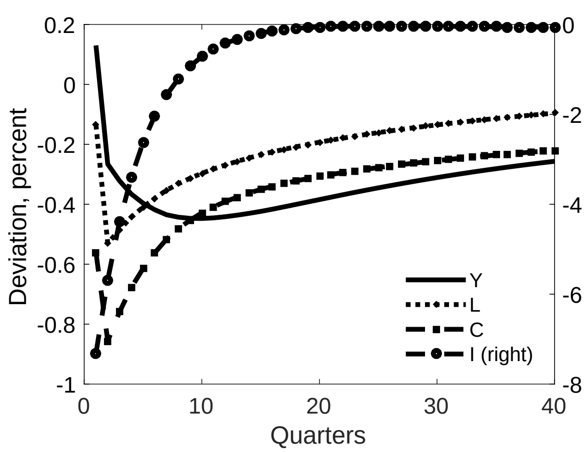
Filtered Shocks and Non-targeted Variables during Crisis

Notes: The figure shows the smoothed shocks fed into the model and compares the evolution of model-generated aggregate variables in response to these shocks to the data. The top-left panel reports the level of smoothed shock series during Chilean sudden stop. The aggregate exogenous shocks across the business cycle are smoothed out using demeaned log differences of aggregate output and interest rate series over 1996:Q1–2011:Q2. Panels b, c, and d report consumption, investment, and hours worked, respectively. The variables are presented in deviations of demeaned log series.

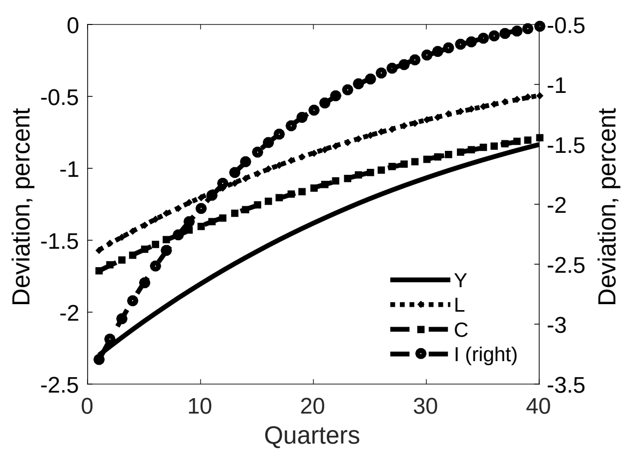
Table XII
Variance Decomposition

	TFP	R
c	0.944	0.056
y	0.969	0.031
L	0.945	0.055
inv	0.453	0.547
a	0.208	0.792
entry	0.262	0.738
ih	0.192	0.808
il	0.281	0.719
vh	0.869	0.131
vl	0.884	0.116

Notes: Consumption, output, investment, and firm values are normalized by endogenous productivity.



(A) Macro Aggregates' Response to R



(B) Macro Aggregates' Response to Z

Figure XV
IRFs to R (left panel) and TFP (right panel) shocks

Notes: The left panel shows the impulse responses of output, consumption, hours worked, and investment (secondary axis) to a one standard deviation interest rate shock, whereas the right panel shows the impulse responses of the same aggregate variables to a one standard deviation TFP shock.

Finally, Figure XVI shows that the exit rate in the model is also consistent with the data. The exit rate is flatter and less volatile than the entry rate.[50]

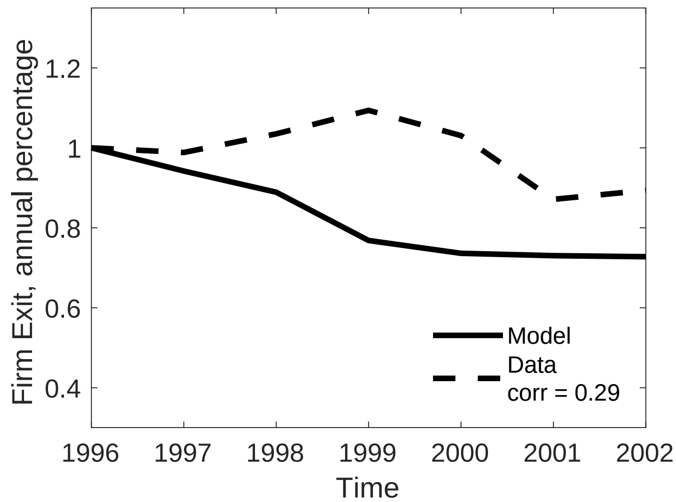


FIGURE XVI
Non-Targeted Exit Dynamics

3.2. Sensitivity Analysis

In this section we discuss the sensitivity of key model-generated moments to the parameters of the model. We focus on three moments (entry rate, share of high-type entrants, permanent productivity loss) and eight parameters (step sizes, $\{\sigma^L, \sigma^H\}$; scarcity, ν; entry cost, κ; parameters of expansion cost function, $\{\varphi, \xi\}$; and working capital constraint parameters, $\{\eta_i, \eta_i\}$). In this exercise, we follow a procedure described in Daruich (2018). Specifically, we draw 100,000 quasi-random Sobol points from an eight-dimensional hypercube, which is defined by a $\pm 1\%$ interval around the calibrated value of each parameter.[51] Then, at each point, we compute the model moments over the business cycle.[52] When doing so, we introduce to the model the shocks that we filtered in the baseline model using the Chilean series of manufacturing output and real interest rate (shown in Figure XIVa).[53]

[50]Incumbent dynamics and aggregate risk are critical for the model to deliver this asymmetric behavior. Without incumbent expansion, the entry and exit rates are necessarily the same, even outside the balanced growth path.

[51]Therefore, if the calibrated value of a certain parameter is x, the interval we consider is $(0.99x, 1.01x)$.

[52]This procedure yields a different BGP at each point.

[53]In this way, we standardize the shock process used and thus avoid additional variation that would stem from filtering shocks again at each point.

We compute the peak deviation in the entry rate and its composition from their respective (recomputed) BGP values. For the permanent loss, we compute the log-deviation in endogenous productivity five years after the sudden stop (similar to the exercise in Section 5.3). Finally, we divide the vector for each parameter into 20 quintiles and compute the 25^{th}, 50^{th}, and 75^{th} percentiles of each moment in each quintile. Figures XVII-XIX plot the variation in each moment along quintiles of different parameters, with blue circles denoting 25^{th} and 75^{th} percentiles of the moment in each quintile, while red circles denote the median value. In a sense, the slope of the median highlights the sensitivity of the moment to the specific parameter, while the difference between highest and lowest quartiles of the moment value in each parameter quintile indicates the relative importance of the other parameters.

A few observations stand out. Step-size parameters have a sizeable effect on all moments around their calibrated value (the calibrated value corresponds to the higher bound of 10^{th} quintile). Second, entrant compositions and permanent productivity loss are strongly sensitive to entry and expansion cost parameters. They are also sensitive to step-size parameters. Third, the scarcity parameter has some effect on the entry rate, but its effect on other moments is relatively muted. Finally, working capital constraint has a negligible effect on the dynamics of the model.

We conclude this section by providing further evidence on the limited role the working capital constraint plays in our mechanism in subsection 3.2.1.

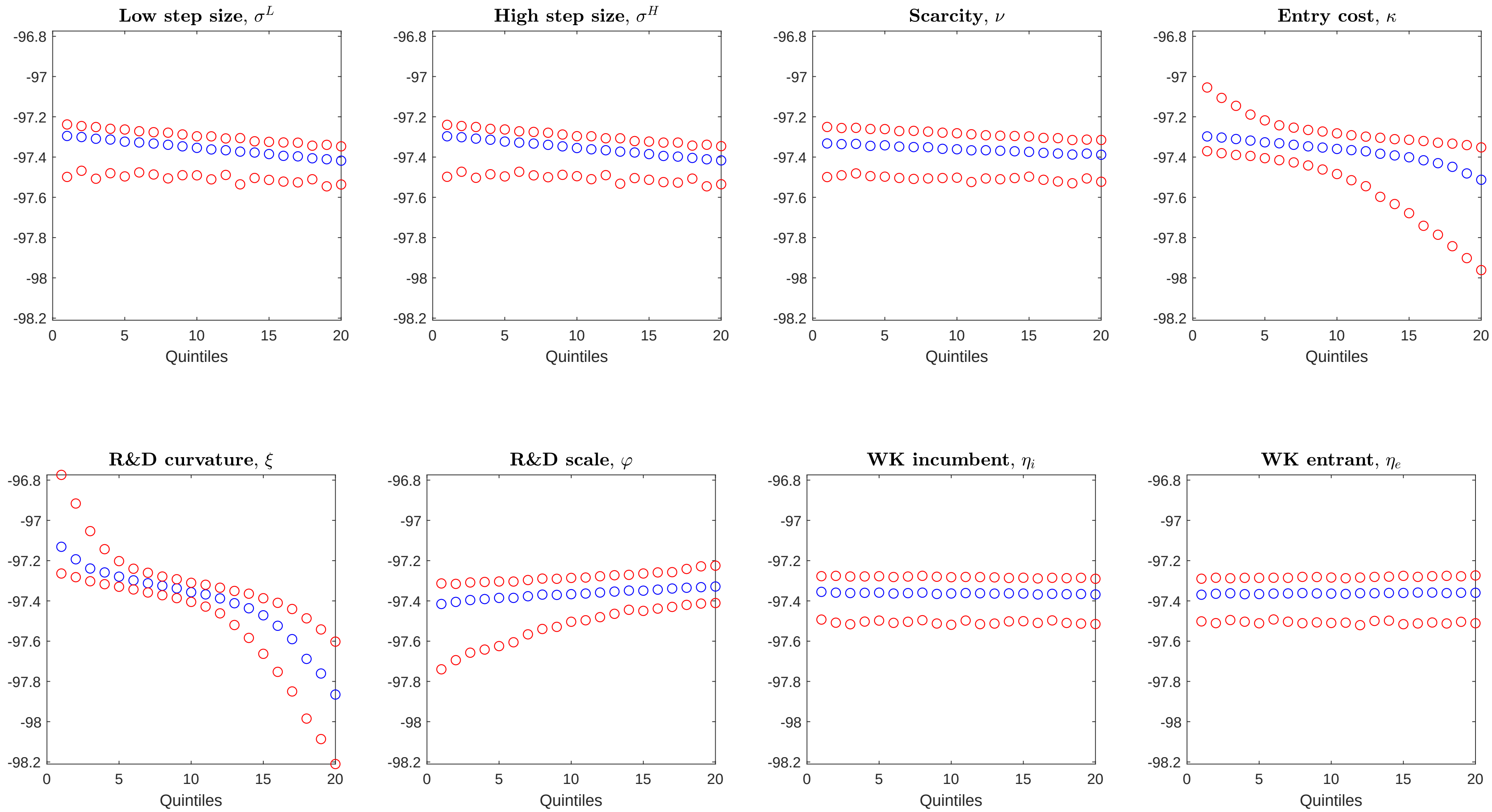


FIGURE XVII
Sensitivity of entry

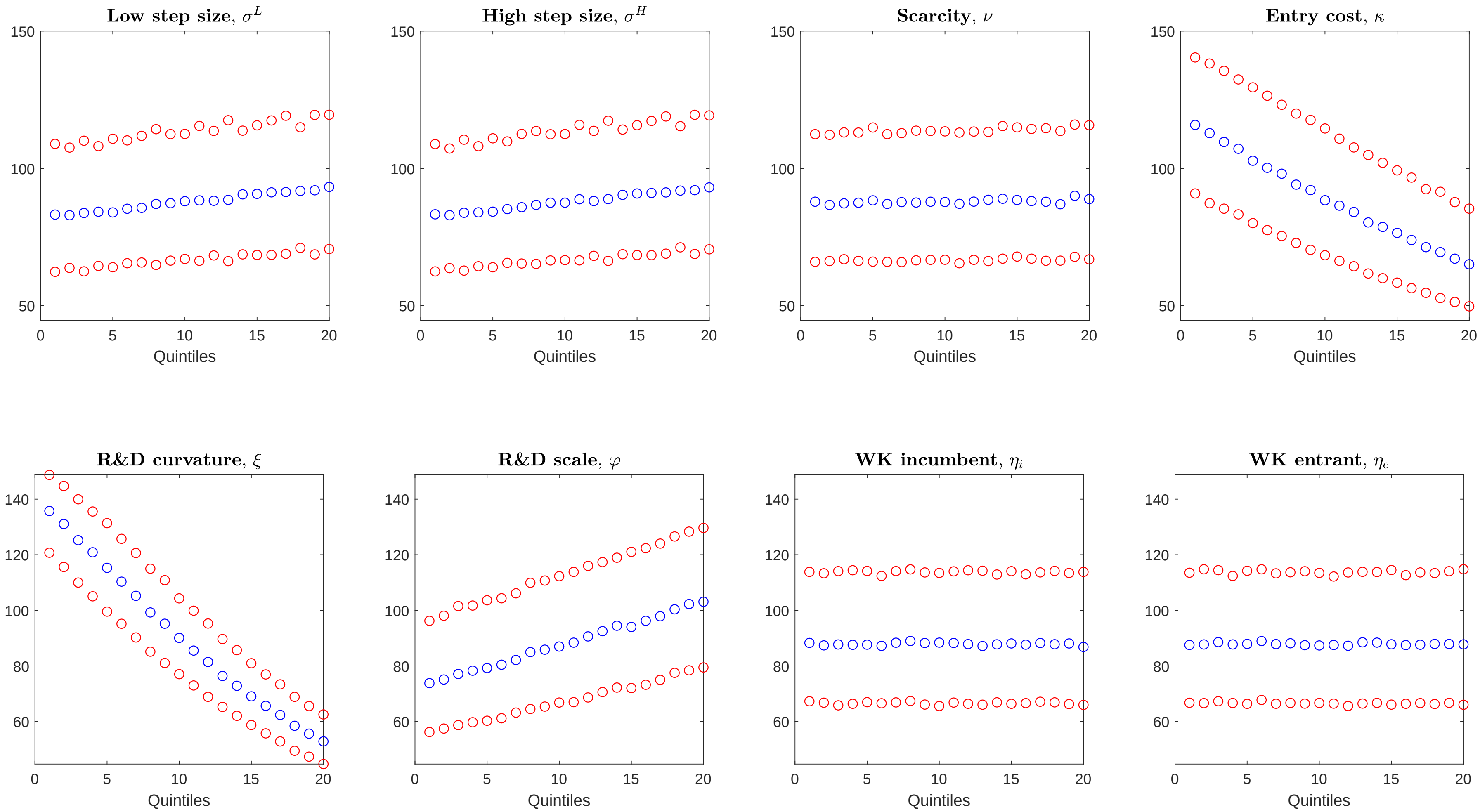


FIGURE XVIII
Sensitivity of entrant composition

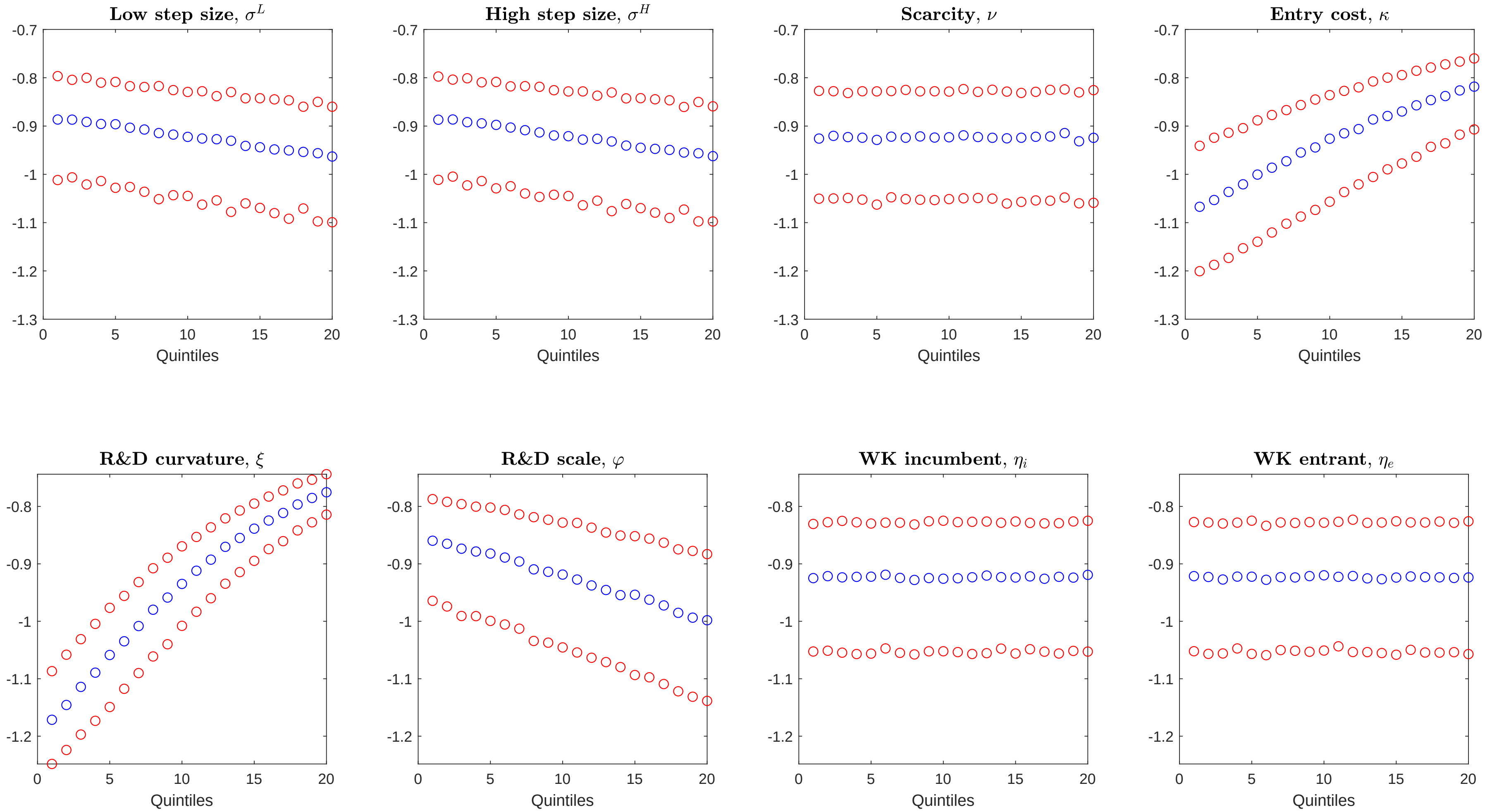
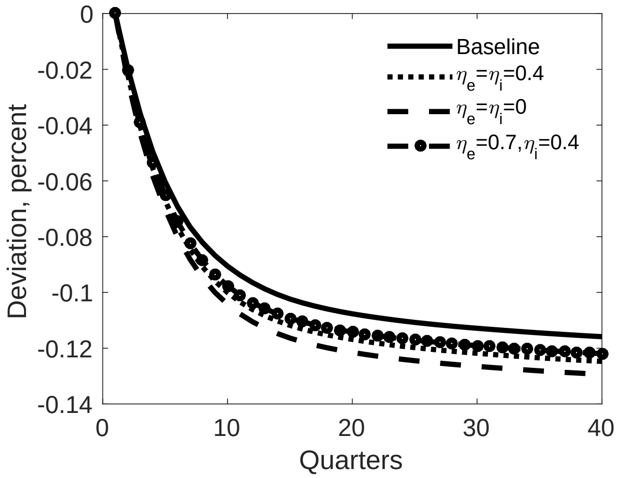


FIGURE XIX
Sensitivity of permanent productivity loss

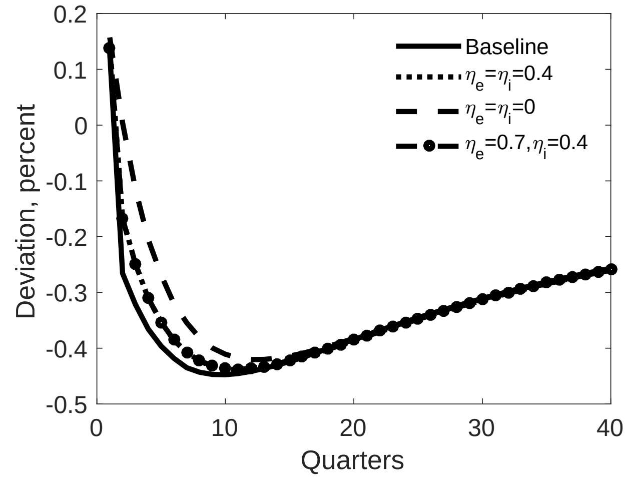
3.2.1. A Further Look at Working Capital Constraint

To explore the role of the working capital constraint, Figure XX compares the baseline calibration of $\eta = 0.6$ to several alternatives. In particular, the dotted line represents an economy with a slightly lower level of working capital needs ($\eta = 0.4$), the dashed line is an economy with no working capital constraint ($\eta = 0$), and the dashed and dotted line represents an economy where the financial intermediary (entrants) face a tighter working capital constraint than the one faced by incumbents ($\eta_e = 0.7$ and $\eta_i = 0.4$). In line with Arellano et al. (2012), the latter economy captures the fact that entrants are likely to be more financially constrained than incumbent firms.

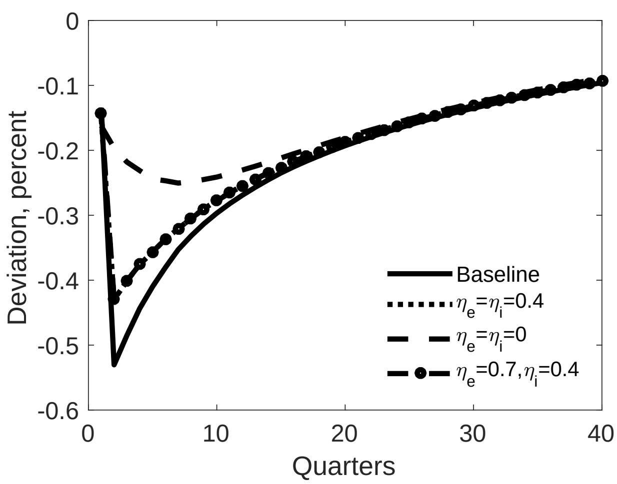
Figure XXa shows that the endogenous productivity component reacts very similarly in every economy to interest rate shocks. Because interest rates are the main driver of endogenous productivity, this similarity implies that our quantification of the permanent productivity loss of the Chilean sudden stop does not depends on the value of η. In fact, Figure XXd shows that every economy predicts the same long-run productivity loss. In contrast, because the working capital channel makes stationary interest rate shocks behave like productivity shocks, we do see a difference in the short-run behavior of output in Figure XXb and employment in Figure XXc. In line with Neumeyer and Perri (2005), the larger the working capital channel is, the stronger the real short-run effects of interest rate shocks are. Interestingly, the economy where entrants are more constrained than incumbents behaves very similarly to the economy where entrants and incumbents are equally constrained. This similarity is driven by the fact that, because entrants have only one product, they therefore account for a very small portion of the economy-wide labor. Finally, this outcome illustrates that the permanent productivity loss of sudden stop is not driven by the working capital constraint but by the effect that the interest rate has on innovation. This effect is driven by the pass-through of interest rate shocks to the value of varieties triggered by fluctuations in the stochastic discount factor.



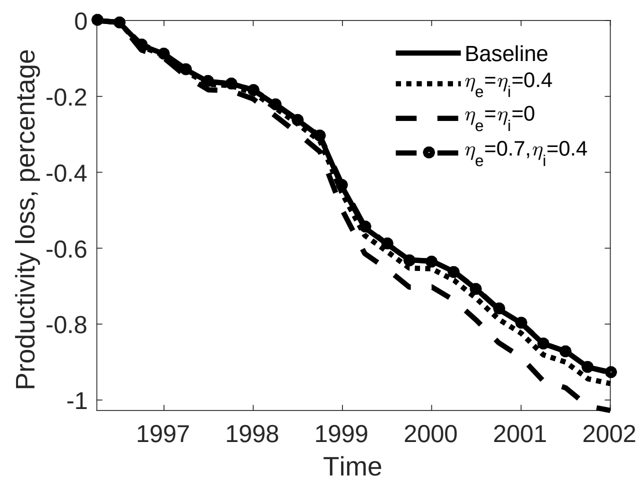
(A) Productivity Response to R



(B) Output Response to R



(C) Labor Response to R



(D) Prodcutivity loss

Figure XX

Impulse response functions to R and long-run productivity cost of the crisis

Notes: The top-left panel shows the impulse response of endogenous productivity growth to a one standard deviation interest rate shock in models with different levels of working capital parameters. The top-right and the bottom-left panels do the same for output and hours worked, respectively. The bottom-right panel shows the percentage loss in the endogenous productivity component over the business cycle with respect to its path along the balance growth path, again across models with different levels of working capital parameters.

3.3. Alternative Models

3.3.1. Model without Heterogeneity (NH)

The model with no heterogeneity eliminates firm types, keeping the expansion decision of firms. This transformation is equivalent to setting $\sigma = \sigma^L = \sigma^H$ in the original model. The following two changes convert the baseline set of equations to the set of equations needed to characterize NH:

1. Any generic variable $\boldsymbol{x}^d$ has a single value; and
2. composition variables in the economy are set to unity, i.e., $\mu = \tilde{\mu} = 1$.

The problem of the financial intermediary is linear and simplifies to a zero expected profit condition:

$$\mathbb{E}\left[m(s^{t+1})\left(1 + a(s^t)\right)\bar{v}^L(s^{t+1})|s^t\right] = \left(1 + \eta\left(R\left(s^{t-1}\right) - 1\right)\right)w(s^t)\kappa. \tag{76}$$

3.3.1.1 Calibration

We want to assess the permanent productivity loss estimated by a model with no heterogeneity. Therefore, we recalibrate the model to match a subset of the original moments. NH has only one step size and no scarcity parameter (ν); therefore, we drop the mean and the standard deviation of the size distribution from the targets and recalibrate the model. The measure of firms is fixed to the calibrated value in the baseline model. Table XIII shows the results of this exercise:

Table XIII
Internally Calibrated Parameters

Parameter	Symbol	Value	Main identification	Target	Model
Labor disutility level	Θ	27.47%	Working time	33.00%	33.00%
Entry Cost	κ	4.62%	Entry rate	10.80%	10.78%
Step Size	σ	7.06%	Annual GDP Growth	3.00%	3.00%
Mass of Varieties	λ	7.62	Mass of Firms	1.00	1.00
Expansion Cost scale	φ	22.12%	Mean of firm employment distribution	7.62	7.61
Stdev TFP	σ_z	1.22%	Quarterly output volatility (HP filtered)	3.00%	3.00%
Capital adjustment cost	ϕ	9.33	Quarterly investment volatility (HP filtered)	9.56%	9.56%

3.3.2. Model without Heterogeneity and Firm Dynamics (NDNH)

The NDNH economy goes one step further and eliminates the expansion decision of firms. In this sense, every firm has only one product, and firms remain in operation until they are replaced by an entrant. Therefore, NDNH is equivalent to NH without ι decision.

3.3.2.1 Normalized System of Equations

The following set of equations represents all the equations of NDNH that differ from the baseline economy.

Final Good Producer

$$y(s^t) = \exp\left(z(s^t)\right) \cdot \left(l_p(s^t)\right)^{\alpha} \left(k(s^{t-1})\right)^{1-\alpha} \tag{77}$$

$$k(s^{t-1}) = \frac{(1-\alpha)y(s^t)}{r(s^t)} \tag{78}$$

Intermediate Good Producers

$$l_p(s^t) = \frac{\frac{\alpha}{\Lambda} y(s^t)}{w(s^t)(1+\sigma)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)} \tag{79}$$

$$\pi_j(s^t) = \frac{\alpha}{\Lambda} \frac{\sigma}{(1+\sigma)} y(s^t) \tag{80}$$

$$\bar{v}(s^t) = \pi(s^t) + \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\left(1-\Delta(s^t)\right)\bar{v}(s^{t+1})|s^t\right] \tag{81}$$

Financial Intermediary

$$\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right) w(s^t)\kappa = \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\bar{v}(s^{t+1})|s^t\right] \tag{82}$$

$$\tilde{\mu}(s^t) = 1 \tag{83}$$

Aggregate Variables

$$a(s^t) = (1+\sigma)^{\frac{M(s^t)}{\Lambda}} - 1 \tag{84}$$

$$\mu(s^t) = 1 \tag{85}$$

$$\Delta(s^t) = \frac{M(s^t)}{\Lambda} \tag{86}$$

$$t(s^t) = \pi(s^t) - \left(1 + \eta\left(R\left(s^{t-1}\right) - 1\right)\right) M(s^t)\kappa w(s^t) \tag{87}$$

$$nx(s^t) = y(s^t) - c(s^t) - i(s^t) - \frac{\psi}{2} y(s^t)\left(\frac{b(s^t)}{y(s^t)}(1 + a(s^t)) - \bar{b}(1+\bar{g})\right)^2 \tag{88}$$

$$d(s^t) = b(s^{t-1}) - \eta w(s^t) l(s^t) \tag{89}$$

$$l(s^t) = l_p(s^t) + \kappa M(s^t) \tag{90}$$

3.3.2.2 Calibration

Compared with NH we drop φ and the share of labor of the 10% largest firms. Table XIV presents the result.

Table XIV
Internally Calibrated Parameters

Parameter	Symbol	Value	Main identification	Target	Model
Labor disutility level	Θ	24.21%	Working time	33.0%	33.0%
Entry Cost	κ	38.48%	Entry rate	10.8%	10.8%
Step Size	σ	32.92%	Annual GDP Growth	3.00%	3.00%
Stdev TFP	σ_z	0.99%	Quarterly output volatility (HP filtered)	3.00%	3.00%
Capital adjustment cost	ϕ	8.89	Quarterly investment volatility (HP filtered)	9.56%	9.56%

Compared with NH, the unique step size is five times larger. This result is due to the fact that the same entry rate needs to trigger the same growth rate but without incumbent dynamics. We can think of the step size in NDNH as a summary of all the innovations that an average entrant on NH would perform during its life cycle.

3.3.3. Model with Exogenous Growth

The economy with exogenous growth is characterized by the same set of equations as the baseline. However, expansion rates (ι^d) and entry mass (M) are taken as parameters, and they are set to the balanced growth path. Thus, the equations that correspond to those variables are dropped from the system. Therefore, by construction, the parameters that determine the BGP of Exo are the same as the baseline calibration. The two remaining parameters—the capital adjustment cost ϕ and the standard deviation of the TFP shocks σ_z—are again calibrated using the business cycle properties the model and take the values $\phi = 9.40$ and $\sigma_z = 1.20\%$. Of note, this model is practically analogous to the economy of Neumeyer and Perri (2005).

3.4. Robustness

In this section, we discuss the robustness of our main results under different model specifications. First, we analyze a version in which R&D is conducted using capital instead of labor. Second, we look at a version in which the entry cost is not linear but convex. Third, we consider a version in which we assume that we observe only a subset of firms in the model economy, reflecting the truncation in the data. Table XV summarizes the calibration results for these three alternative specifications. Table XVI presents the long-run loss and consumption-equivalent welfare changes in response to a 100-basis-point shock to the interest rate in each version. We will refer to these tables when discussing each version in more detail below. To summarize briefly, the results in this section show that the main findings in the baseline version go through under these robustness specifications.

TABLE XV
INTERNALLY CALIBRATED PARAMETERS

Parameter	Symbol	K in R&D	Quadratic entry	Truncated	Main identification	Target	K in R&D	Quadratic entry	Truncated
Mass of Varieties	λ	7.06	6.96	−−	Mass of firms	1.00	1.01	1.12	−−
Unaccounted employment	λ	−−	−−	7.11	Unaccounted employment	33.00%	−−	−−	33.43%
Labor disutility level	Θ	30.56%	30.67%	29.17%	Working time	33.00%	31.27%	31.92%	33.00%
Entry Cost	κ	31.06%	85.08%	5.00%	Entry rate	10.80%	12.78%	13.88%	7.84%
Step Size H	σ^H	6.36%	6.56%	7.97%	Annual GDP Growth	3.00%	3.01%	2.81%	3.10%
Step Size L	σ^L	6.27%	6.20%	7.35%	Mean firm employment	7.62	7.00	6.23	7.53
Scarcity	ν	55.57	54.62	73.28	Stdev firm employment	13.29	12.84	12.38	13.30
Expansion Cost scale	φ	13.35%	20.99%	30.97%	L-share of top 10% firms	48.30%	52.26%	55.58%	50.14%
Stdev TFP	σ_z	1.17%	1.22%	1.22%	Output volatility	3.00%	3.00%	3.00%	3.00%
Capital adjustment cost	ϕ	9.34	9.52	9.28	Investment volatility	9.56%	9.56%	9.56%	9.56%

TABLE XVI
LONG-RUN OUTPUT AND WELFARE COST OF A 100 BPS R SHOCK

	Baseline	K in R&D	Quadratic entry	Truncated
LRC	-0.24%	-0.33%	-0.19%	-0.21%
LRC rel. to Baseline	100%	135%	78%	88%
CEQ	-0.15%	-0.17%	-0.14%	-0.13%
CEQ rel. to Baseline	100%	113%	93%	91%

Notes: LRC and CEQ stand for long-run cost and consumption equivalent welfare cost, respectively. A negative x% for LRC means that the endogenous productivity is x% lower than the un-shocked path in the corresponding model 1200 periods after the shock hits. A negative x% for CEQ implies that the representative household in the shocked economy would have the same welfare if she consumed x% less in the un-shocked economy. The "K in R&D" model refers to a version in which entry costs and productivity-enhancing investment by incumbents are quoted in physical capital instead of labor. The "Quadratic entry" model refers to a version in which the entry cost is quadratic in entrant mass. The "Truncated" model refers to a version in which entry is defined as expansion of a firm that has one product line, to account for the employment cutoff in the data.

3.4.1. Capital Input in R&D

In this exercise, we analyze a version where productivity enhancing investments require capital input instead of labor, bringing the specification of investment closer to the standard small open economy model. Specifically, enacting a new project requires $\kappa A(s^t)$ units of capital. Therefore, the cost of enacting a measure of $M(s^t)$ projects is given by

$$cost(M(s^t)) = r(s^t)\kappa A(s^t)M(s^t).$$

Similarly, the cost of expansion per product line at rate ι for a d-type incumbent is given by

$$cost(\iota^d(s^t)) = \varphi r(s^t)A(s^t)\iota^d(s^t)^\xi.$$

We recalibrate the model, and the results are shown in column 1 of Table XV. As highlighted in column 4, the match is quite successful only with the entry rate being somewhat above the target. The BGP capital to output ratio, which is not targeted in any calibration exercise, is 2.7 in this specification compared with 2.4 in the baseline case. We then assess the implications of the model for the long-run loss in productivity and consumption-equivalent welfare loss in response to a 100 basis points interest rate shock, similar to the analysis in Section 5.4. As shown in column 2 of table XVI, the findings are in line with the baseline, only with some larger long-run loss in this version. Therefore, we conclude that our findings are robust to this alternative specification.

3.4.2. Quadratic Cost of Entry

In this specification, we consider a quadratic cost for entry, which makes the entry cost and incumbent R&D cost functions to have the same convexity. We assume the following specification for entry cost:

$$cost(M(s^t)) = \frac{\kappa}{2} M(s^t)^2 \bar{W}(s^t).$$

The calibration results for this version are shown in column 2 of Table XV. As shown in column 5, the match is fairly good, with the calibration missing the mean and the standard deviation of the employment distribution somewhat on the downside, while the entry rate is above the target. Turning to Table XVI, column 3 reveals that in this case, the implied long-run loss is a bit smaller than in the baseline. This result would be expected, as the convexity in the entry cost damps the adjustment in the entry margin, limiting the permanent loss. However, the welfare loss generated by the interest rate shock is very similar to that in the baseline. The reason is that a smaller drop in entry relative to the baseline means fewer resources being diverted to production from entry activity. But overall, the implications of this specification again echo our baseline findings.

3.4.3. Employment Cutoff in Data and Truncation

In ENIA, we observe firms that have at least 10 workers. To reflect this truncation in the data, we consider a version of the model in which we assume that in the model economy we observe only firms that have at least two product lines. As such, we truncate our model economy in a similar way.

While, in contrast to the previous two exercises, this specification does not alter the equations that define the equilibrium; it changes how we compute some of the moments in the model. In particular while aggregate moments such as the growth rate and working time are computed based on the whole economy, entry rate and moments regarding the employment distribution are computed based on the truncated sample of firms. Crucially, we define entry in this economy as obtaining at least two product lines. Moreover, in line with the empirical analysis, we carefully keep record of firms that lose product lines and shrink to only one line in order to not recount them as entry in case they successfully expand again. In other words, we distinguish first-time entry from reentry.

The recalibrated values of the model parameters are listed in column 3 of Table XV.

Of note, in the recalibration, we replace the moment "mass of firms" with the share of employment that is not employed in the truncated model economy. The empirical counterpart of this moment is roughly a third of the labor force. In this way, we discipline the size of the truncated economy based on the data. The overall match to the data is quite good, except for a somewhat low rate of entry. Also, the last column of Table XVI reveals again that the main findings of the baseline model are intact in this version. Therefore, we choose to proceed with the baseline model, refraining from additional complexity that this specification creates.

3.5. Firm Size and Crises

Sedláček and Sterk (2017) use U.S. census data to document the cyclicality of job creation by startups and the persistence of these differences over the life cycle (at five years in the baseline result). Because our mechanism suggests that, on average, better cohorts with proportionally more H-type firms—which have higher growth potential—enter during downturns, our model may appear to contradict these findings, making it worth discussing our model's implications in this regard. To shed light on this point, Figure XXI shows the employment levels (in percentage deviations from the respective value of the first cohort) of startup and five-year-old cohorts, where the time series is shifted back to the year of their birth.[54]

Our baseline economy predicts a cyclical variation in the size of startup cohorts, and this feature persists even at age five, echoing the findings in Sedláček and Sterk (2017).[55] Expansion rates (ι^d) are common to every firm of type d regardless of its size or age. Moreover, ι^d is procyclical. Therefore, a d type firm with age T will be larger in expectation if most of those T years were expansions. However, firms born in 1997 spend most of their early years during crisis, and therefore, are the smallest at age five, according to the model.

Potentially, the composition effect could be strong enough to overcome this force. In fact, high-type firms always expand faster than low-type firms; then, if cohorts born during crises have more high-type firms they could end up larger on average. However, the composition of cohorts born during booms and downturns is different mostly at very young ages,

[54] The exercise replicates Figure 1 in Sedláček and Sterk (2017).

[55] When we perform the empirical analysis on equation (37) using labor growth rate as the dependent variable we see that firms born during crises do not grow more quickly. Interestingly, when we use physical investment as a measure of growth, we see that firms born during the crisis accumulate capital more quickly. Because this analysis is beyond the scope of this paper, those regressions are available upon request.

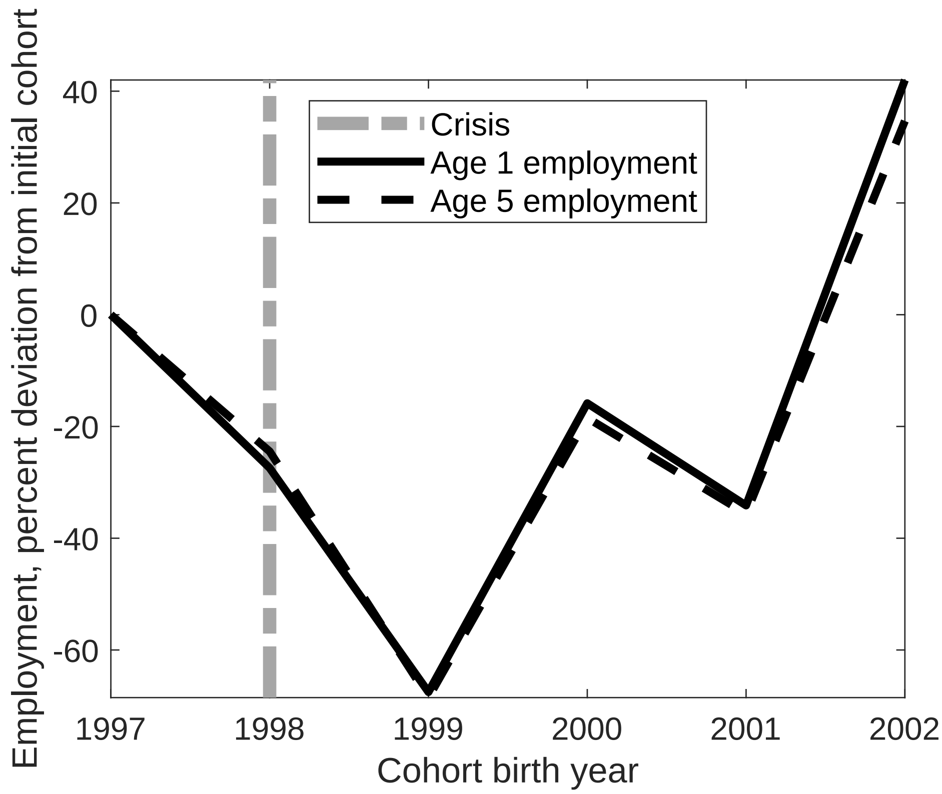


Figure XXI
Average Cohort Employment 5 Years after Entry (relative to first year)

and as the cohorts get older, the proportion of high types rises for both type of cohorts because low-type firms are scrapped more quickly. This relationship reduces the compositional differences at birth across cohorts over time and drives the result in Figure XXI, as the exercise considers cohorts at already five years into their life cycles. Therefore, this model can generate the basic premise documented by Sedláček and Sterk (2017) for the U.S. economy. Future research should explore a closed economy version of our economy and compare it to the U.S. firm-level dynamics.